VIBRATIONS OF PACKETTED TURBINE BLADES

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CERTIFICATE

This is to certify that the thesis entitled "VIBRATIONS OF PACKETTED TURBINE BLADES" is a record of work carried out under my supervision and that it has not been submitted elsewhere for a degree.

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2.8.72

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TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

NOMENCLATURE

a,b	:	Dimensions of the rectangular blade cross-section
a	:	A square Matrix which relates the local coordinates in terms of nodal displacements.
b	:	Square Matrix relating strains and displacement vectors
B	:	A constant (See equation 2.18)
C1	:	Torsional Stiffness constant considering Saint - Venant - Resistant to shear
C2	:	Torsional Constant due to bending of longitudinal fibres
C3	:	Torsional Constant to take into account twist of the rod
C.G	:	Centre of Gravity
D	:	Dynamical Matrix of the system
E	:	Young's Modulus
f	:	Natural Frequency
f_i, f_n	:	i^{th} or n^{th} natural frequency
F	:	Centrifugal force due to an elemental force
F_y, F_z	:	Components of F
F*	:	Equivalent continuous body forces expressed in discrete system
F'	:	A constant vector, being independent of co-ordinate displadements
G	:	Modulus of Rigidity
$I_{c.g.}$	:	Moment of inertia of the [illegible]
I_p	:	Polar moment of inertia
I_{yy}, I_{zz}	:	Moment of the inertia of the section about y and z axes

I_{yz} : Product Moment of inertia about y and z axes

J_x : Mass moment of inertia about X axis

K : Stiffness matrix of the system

K^{-1} : Inverse of the matrix K

K_i : i^{th} correction matrix used to account for the centrifugal forces

$\overline{K}$: Corrected stiffness matrix

K_n : Southwell Constant for n^{th} frequency

K.E. : Kinetic energy

K_t : Additional K.E. due to torsion

l : Length of the element

L : Length of the Blade

M : Mass of the band of length equal to the blade span length

M : Mass matrix of the system

m : Mass of the blade per unit length

m.n : Order of matrices

n : Number of blades

N : Speed of Rotation (r.p.m.)

p : Natural frequency

p_{rn} : n^{th} natural frequency for rotating blade

p_{Nn} : n^{th} natural frequency for non rotating blade

PE : Potential Energy

Q : Generalized discrete forces acting at the nodes

r : Distance between the elemental mass of a rotating blade and the axis of rotation

R : Radius of the inner end of the element

R_O	:	Root radius of the blade
P	:	Concentrated forces at the node points
p_s	:	Generalised forces equivalent to surface traction expressed in the nodal coordinates
p_b	:	Generalised forces equivalent to continuous body force
S	:	Surface of the element
v	:	Volume of the element
V_t	:	Additional kinetic energy contributed due to torsion
V.R.	:	Vibration per revolution
x,y,z	:	Coordinates of the rotor-blade system
X,Y,Z	:	Principal axis of the corss-section
y', z'	:	Slopes in the y and z direction with respect to x
$\dot{y}$, $\dot{z}$	:	Derivative with respect to time
w	:	Density of the material
U	:	Generalized co-ordinate system
$\ddot{U}$	:	Second derivative of U with respect to time
r_z, r_y	:	The distance between the C.G. and the centre of twist, measured along the z and y axes.
α	:	Twist per unit length
ξ	:	Band to blade stiffness ratio
ξ	:	x/L
θ	:	Angular displacement of the blade
ω	:	Circular natural frequency
γ	:	Specific weight
ρ	:	Specific mass

ϕ : Surface forces

ϕ : Stagger angle

χ : Eigen values

Ω : Angular velocity of the turbine rotor

SYNOPSIS

of the
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In the present work, the problem of packetted H.P. Turbine Blades is analysed to study its vibrations i.e. to calculate the fundamental and a few higher natural frequencies and to find the corresponding mode shapes. Packetted Turbine Blades are made of a number of individual blades grouped together by a rivetting a steel band at the free end.

Two modes of vibrations, tangential and coupled transverse - torsional modes are considered. The effect of number of blades in a group and that of band to blade stiffness ratio on the natural frequencies of vibrations are considered. Finite Element method is used in the analysis to obtain consistant mass matrix and corrected stiffness matrix. From these Dynamical matrix of the system is obtained. Its eigen values and eigen vectors give natural frequencies and the mode shapes. The

stiffness correction matrices are derived to take into account the effect of centrifugal forces due to the blade and the band in both the modes.

Results are plotted such that the characteristic curves give the variation of natural frequencies as a function of number of blades in the group for different band to blade stiffness ratios. These graphs are to enable the designer to avoid a certain frequency of vibration by proper selection of band properties and / or the number of blades to be packetted.

CHAPTER I

INTRODUCTION

It is commonly observed that most of the turbine blade failures occur due to fatigue at the blade roots. Reliability of operation of steam turbine is of prime importance which requires that in design state the mechanical reliability of the blade be given a major consideration. The fatigue failure of the blade is usually associated with the excessive vibrations in the neighbourhood of its resonance frequency. For the safe design the designer makes sure that the natural frequencies obtained by theoretical calculations are reasonably away from the forcing frequencies.

To consider the vibrational analysis of the turbine blade, some of its important features are presented below:

a) Configuration of the turbine blading
   - i) Shrouding
   - ii) Lacing Wires

b) Excitations involved in the operation of turbines
   - i) Internal
   - ii) External

c) Tunning Process, and

d) Design Considerations of blade.

1.1 Configuration of Blades

The steam turbines are generally divided functionally in three parts, namely High Pressure (H.P.), Intermediate Pressure (I.P.) and Low Pressure (L.P.) The blades are fixed on the rotor at the roots and the other end is called the top or free end. The blades used in these sections are discussed below.

1.1.1 High Pressure Section (H.P. Section)

In the H.P. Section, the specific volume of the steam is very low and its expansion is not much. Because of these flow conditions, blades used are of short length which is of the order of a few inches. The blade is small in comparison with the root - radius, (1 : 20) giving the same tangential velocity throughout its length. From the flow requirements, the entrance and exit angles at all points along the blade length are same, and the blade is straight and without any twist.

Shrouding

For the economical use of the steam, having high density, the steam leaking past the blade tip is avoided by rivetting a band strip on the free edges of a blade/a group of blades, see figure 1.1. This prevention of steam leakage

is particularly important in the reaction turbines where a pressure gradient exists across the moving blades. In the high pressure impulse turbines the partial admission is employed. The band is beneficial in stiffening the blades.

The shrouding connecting the blade tops may be continuous throughout the periphery of the rotor or in segments integral with one or more blades. The shrouding is usually attached to the blades by means of tenons cast or forged integral with the blades. Tenons fit through the holes in the shroud and are rivetted into small spherical surfaces. Usually the continuous band is broken into many groups of blades with a gap of the order of 0.03". This is allowed to account for its expansion. The band which runs continuously over the periphery, has a tendency to distort under the influence of high temperature and may break away from the tenon causing eventually the blade failure.[2]

1.1.2 Intermediate Pressure Section (I.P. Section)

When the steam passes through I.P. Section the steam expands successively. To accommodate this increasing volume, the steam passage is made wider and wider. This essentially requires providing larger and larger blade lengths along the steam flow direction.[2] The blade height is a function of the total annular area required to pass the steam flow, which in turn is a function of mass rate flow,

specific volume, and velocity ratio. Due to longer blades in the last ring of the I.P. Section the blades are slightly twisted. The blades still need shrouding. The lengths of the blades are of the order of 4 to 6 inches.

1.1.3 Low Pressure Section (L.P. Section)

In this portion the specific volume becomes very large. The blade lengths are of the order of 10 to 12 inches. The blades become appreciably long and as such entrance and exit blade angles vary along the blade lengths. For this it requires that the blade should be twisted. The blades are also made tapered by cutting down the blade mass at the ends so that the level of stresses due to centrifugal forces at the root of the blades is reduced.

Lashing Wires

One of the reasons for using shroud is to hold blades in the correct position even under the influence of steam forces. This is also achieved by Lacing or Lashing Wires. Shrouding introduces high centrifugal forces where as lacing achieves the purpose of alignment without appreciable addition to centrifugal forces.

Formerly lashing wires were strung through the bossed holes, machined in the blades. The present practice is to weld short pieces of lashing wires to each side of the

blade. When the blades are machine - finished and stresses are relieved, these lashing wires touch each other and are joined either by a weld or by a welded sleeve. The main disadvantage of lashing wires is that it disturbs the flow pattern and contributes to the vibration tendencies. To avoid these, ~~aerofill~~ air foil section is used for the lashing wires. Lashing wires could be provided at two or three places along the length of the blade in order to give it sufficient rigidity.

1.2 Excitation Forces Developed in the Turbines

Having presented the configuration of the blades, we present below different types of disturbing forces developed in the steam turbine. The fundamental forcing frequency imposed on the blade is equal to the turbine rotor speed where as its higher harmonic excitation frequencies are due to varying steam flow from nozzle to nozzle. However these exciting frequencies are not necessarily same for different stages of the turbine. Different kinds of exciting forces are given below.

1.2.1 Internal Excitations:

In stationary steam turbines the main source of excitation is the steam itself. Even in complete circumferencial steam admission, the steam flow is not uniform due

to the following practical structural features (3)

(a) The runner blades experience steam flow of varying velocity as these pass across a stationary guide vane, having maximum velocity in the central zone of the stream and minimum velocity at the portion issuing out from the neighbourhood of the walls of the nozzle, see figure 1.2. This velocity variation imposes a forcing frequency given by,

$$f = \frac{n.N}{60} \text{ c.p.s} \tag{1.1}$$

where n = no. of nozzles for a ring

N = speed of the rotor in r.p.m.

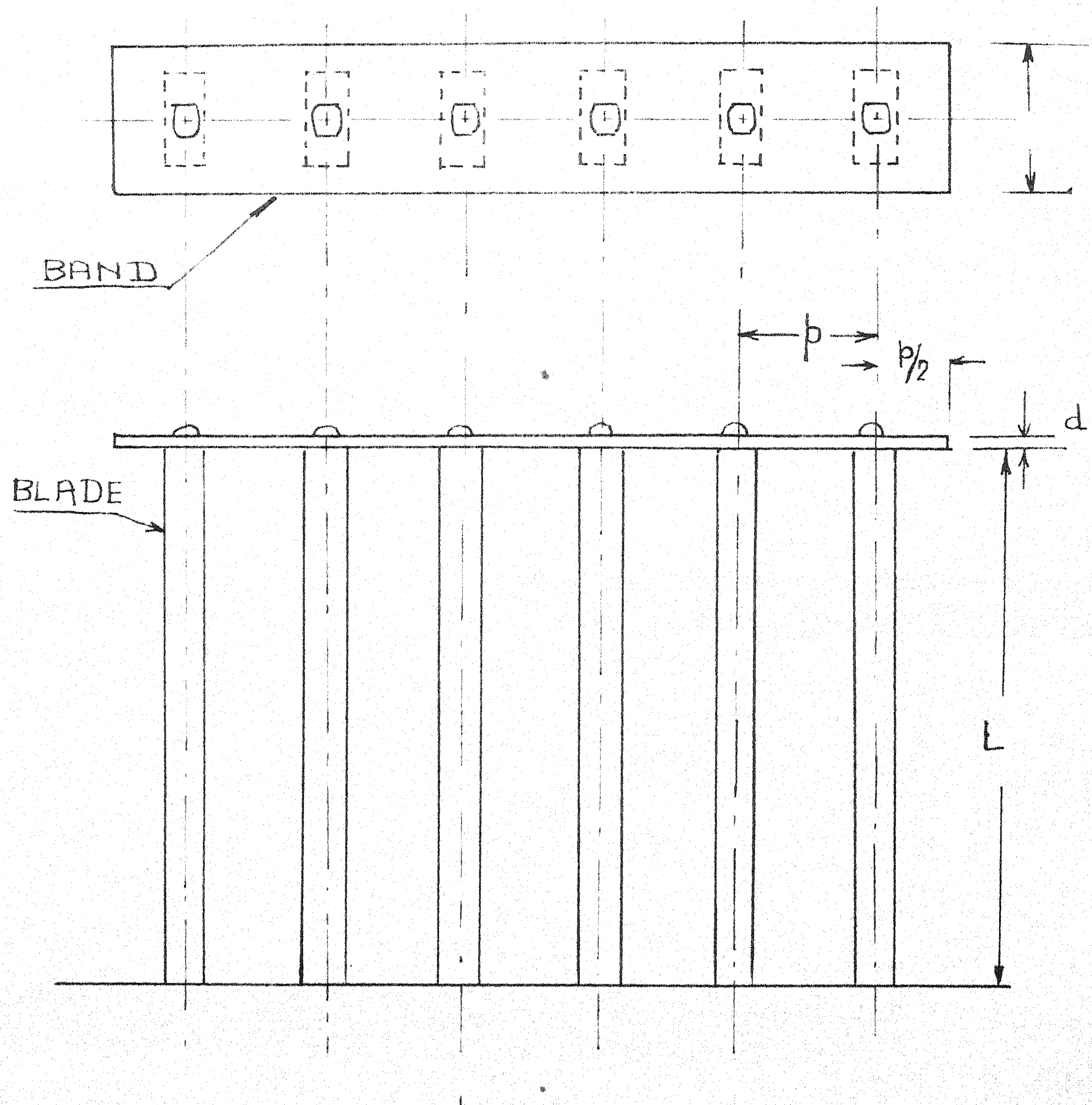
f = forcing frequency

(b) Several discrete steam inlet pipes

(c) Discrete steam extraction pipes

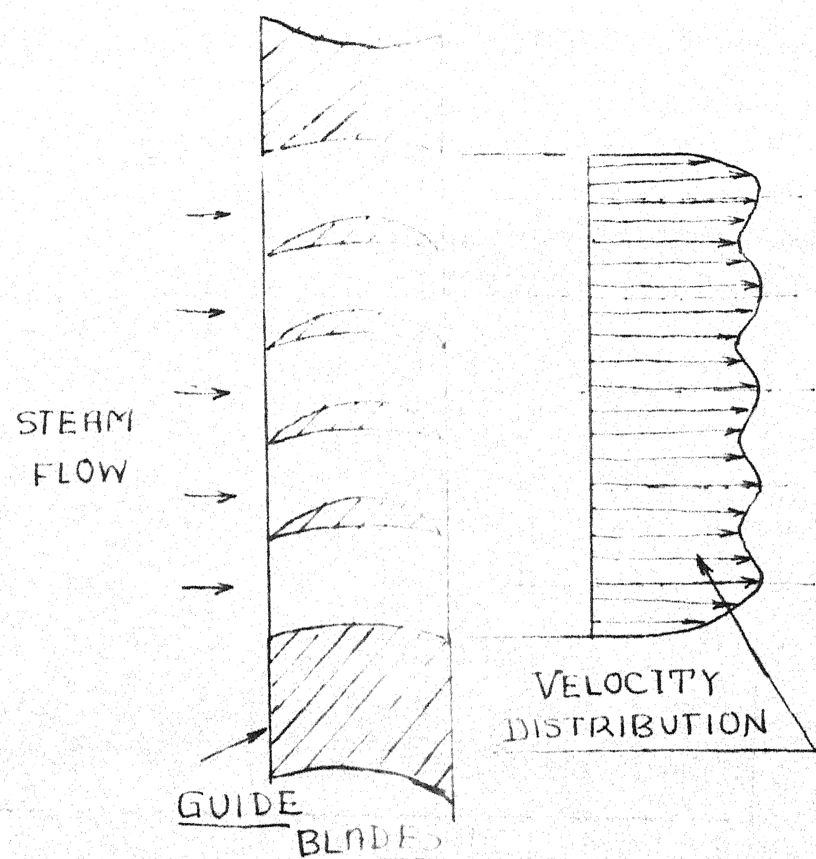
(d) Lack of uniformity in the nozzles or guide blades may also become a source of uneven forces. Due to discontinuity at the horizontal joint, it is difficult to keep the nozzles at the same spacing. Because of the changed positions of nozzles at the horizontal joint, the runner blades experience additional disturbing forces twice per revolution.

(e) Structural members in the inlet and exhaust pipes



BANDED GROUP OF
H.P. BLADES

Fig 1·1



STEAM FORCES

FIG 1.2

CAMPBELL DIGRAM :

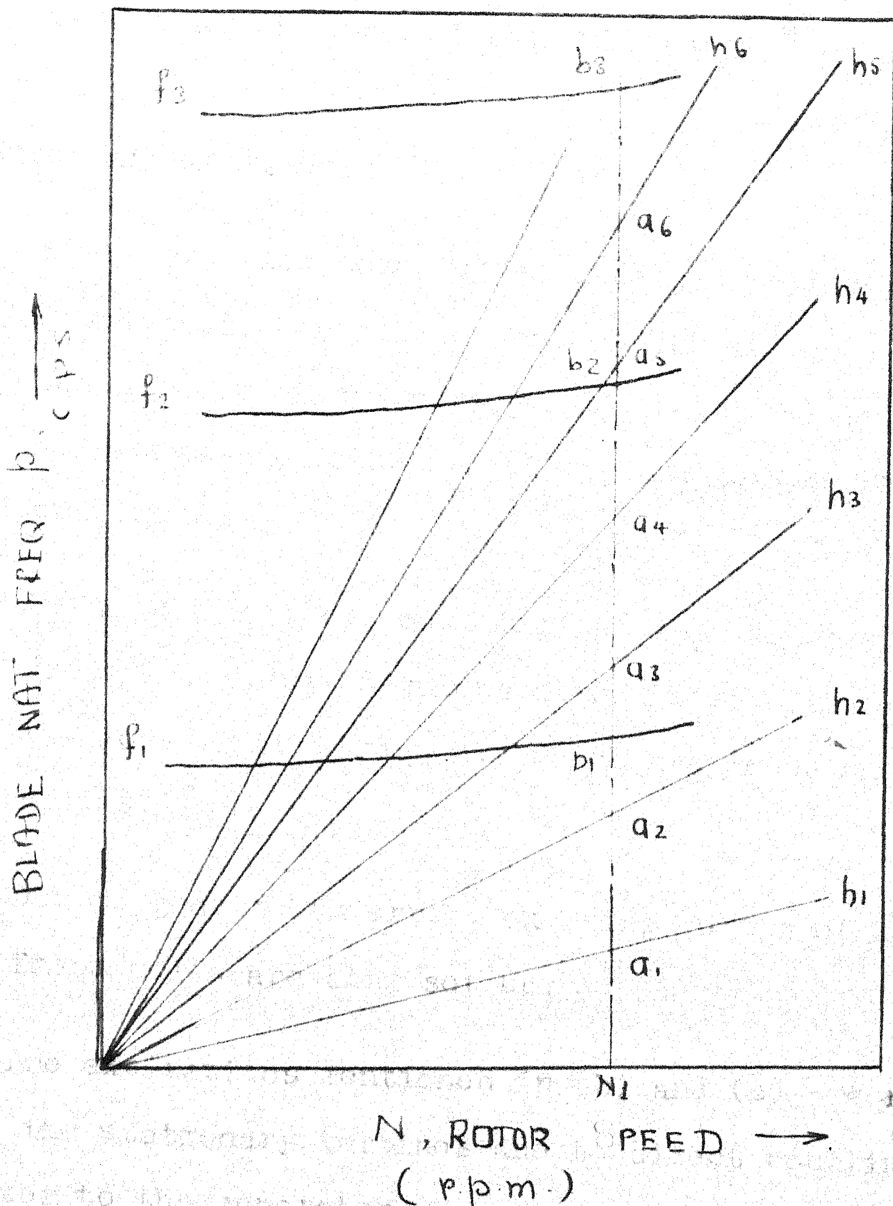


FIG 1·3

(f) Manufacturing variation in the nozzles and blades

(g) Impulse Excitation

this occurs in the H.P. stage due to partial admission of steam and the blades experience sudden application of the full force.[4]

1.2.2 External Excitations

There are also some forces external to the rotor which cause excitation and these are as follows :

(a) Due to lack of cylimdrical nature of the journal. This causes a motion which may set blades in vibrations.

(b) In marine applications, the turbine - gear - propeller system develops strong torsional oscillations of low frequency which are imposed on the blades.

(c) Due to gear tooth error, exciting forces of higher frequencies are also set up.

The excitations mentioned in (b) and (c) are not present in the stationary turbines due to direct coupling of turbine rotor to the generator.

1.3 Tunning Process

Tunning process is a process of varying physical parameters of a blade like length, breadth, thickness and its taper. This may be done by using a parameter or a combination of parameters subject to certain constraints. Tunning is necessary in order to set a particular natural frequency. This is carried out after the blade has been designed from the flow and strength considerations. In the case of banded blades this can be achieved by additional means, such as varying the geometrical properties of the shrouding or varying the number of blades being grouped.

The tunning process is conveniently carried out by means of Campbell diagram named after late Wilfred Campbell[5] of G.E.C.

In the Campbell diagram,[3,4,5] important harmonic frequencies, (usually cycles per minute) are plotted against turbine revolutions per minute, see figure 1.3. These harmonics are represented by straight lines n_i passing through the origin with their slope equal to their ith order of harmonic. In addition to these the natural frequencies of the blades f_i are plotted on the same graph. The intersections of these two sets of lines f_i and h_j indicate the resonance.

With the help of this diagram, the designer avoids the coincidence of important natural frequencies f_i, (fundamental or higher) with the forcing frequencies of excitations. Ideally, the intersecting point a_i of the harmonic h_i and the rotor speed N should lie symmetrically about the intersections b_j and b_{j+1} of natural frequency f_i and the rotor speed ordinate N.

Vibration per revolution, denoted by VR, is a term defined as the ratio of blade frequency f to the rotational speed of the rotor N. This may or may not be an integer. Vibrations per revolution VR of 3 should be avoided because it means that the forced frequency has its third harmonic h_3 coinciding with one of the f_i; similarly VR = 4.9 means that fifth harmonic h_5 is very near to one of the natural frequencies of the blade and should also be avoided. Vibrations per revolution of 2.5 means that the intersection points a_2 and a_3 are equally away from b_i and b_{i+1}. Hence the values of VR 2.5, 7.5 etc. should be ideally achieved.

For short blades centrifugal action has negligible effect on natural frequencies f_i's. Whereas for long blades it is prominent. Thus the natural frequency of the blade is independent of N for short blades and depends upon N for long blades. In the tunning process the straight lines/ curves for f_i are either raised or lowered so as to obtain the ideal VR.

1.4 Design Considerations of Blade

The blade is primarily designed from the flow and strength considerations. The forces acting on the blade are of two types namely static and dynamic loading.

1.4.1 Static Loading

The static loading on the blade is due to steam pressure and the centrifugal force. In general, these forces are high but fortunately these are of static nature and do not cause difficulty in designing for the same.

1.4.2 Fatigue Loading

Several turbine accidents have been directly attributed to blade vibrations caused by synchronism of certain periodic disturbing forces with the natural frequency of the blade. Hence one must be particular to take into account these vibratory forces to avoid the possibility of fatigue failure.

As mentioned before, the blades are subjected to the vibrations caused by irregularities in the fluid flow, non-symmetry of stationary guide vanes, radial pressure distribution, shock and partial admission, etc. A similar effect is also produced in the gas turbines due to seperate combustion chambers.

Lashing wires or shrouds add stiffness to the blades and in some measure these can be used to change the natural frequency of the group. We have seen that the blade configuration varies from stage to stage in a section. Thus blades in different rings have different sets of fundamental and higher natural frequencies. Similarly the number of nozzles per ring may vary from stage to stage and here too we get a spectrum of forcing frequencies. It follows that for a given rotor speed, blades in one or more than one ring may run with natural frequencies equal to or in the neighbourhood of the forcing frequency. In case of constant speed stationary turbines, it is possible to tune blades of all the stages.

It is difficult to achieve the same in case of variable speed marine turbines. For these, tunning process is carried out for the maximum operating rotor speed so as to keep the vibratory forces at the minimum even though the centrifugal forces are high. For any other operating speed, the resultant forces are within the designed limit.

CHAPTER II

REVIEW OF THE PREVIOUS WORK

Turbines are generally divided functionally in three different parts namely H.P., I.P. and L.P. Sections. In the previous chapter we have mentioned about the types of blades used at different stages of the turbine. Blades used for H.P. section are short and straight and are always bonded. Blades in I.P. section are of medium heights, sometimes slightly twisted, and are usually banded. The blades in the L.P. section are long, tapered and twisted. Generally these blades are held together by lashing wires. We present here a short review of the work done in the area of vibration of turbine blades, with particular emphasis on tapered twisted blades and banded H.P. blades.

William Carnegie and J. Thompson [6] considered the effect of width taper on frequencies and mode shapes of straight rectangular blade. This cross - section is selected to isolate the effect of taper from those of parameters like twist and nonsymmetry of section. Width taper is varied over a wide range, from - 0.07 to + 0.1 inch per inch of blade length. Finite difference method is used for the derivatives which reduce the equations of motion to a set of algebraic simultaneous linear equations which forms an eigen value problem. The results obtained from the eigen

value problem are comparable with the experimental results. The authors concluded that the natural frequencies decrease with the increasing positive width for all modes of vibration and the change in frequencies is more sensitive to negative range of values.

Rosard D.D[7] considered the natural frequency of twisted blade of rectangular cross - section with particular interest to investigate the effect of width to thickness ratio. In general, when two systems of nearly equal natural frequencies ω_1, ω_2 $(\omega_1 < \omega_2)$ are coupled, the natural frequencies f_{c1}, f_{c2} $(f_{c1} < f_{c2})$ of the coupled system are such that $f_{c1} < \omega_1$ and $f_{c2} > \omega_2$. In the case of beam vibration problem the two vibrating systems involved are the second beam mode vibration in the flexible direction and the first mode vibration in the stiff direction. The coupling is introduced by the twist of the beam. The author found that the coupling becomes appreciable for the width to thickness ratio in the range of 4 to 12. The problem is analytically solved by the lumped mass system and by applying Myklestad Method[8]. The values obtained are compared with the experimental results and the errors involved are in the range of 0.1 to 2 per cent.

B. Dawson[9] has used Rayleigh - Ritz method to calculate the natural frequencies of coupled bending -

bending vibrations of pre-twisted cantilever beam. The beam of aerofoil section is considered. The expressions for kinetic and potential energies are derived and are as follows,

$$PE = V = \int_L \left[\frac{E\, I_{zz}}{2} (y'')^2 + E\, I_{xy}\, y''\, z'' + \frac{E\, I_{yy}}{2} (z'')^2 \right] dx \qquad (2.1)$$

$$KE = T = \frac{1}{2g} \int_L (\dot{y}^2 + \dot{z}^2)\, dx \qquad (2.2)$$

The deflection curves y (x), z (x) are assumed in a way to satisfy the geometric boundary conditions and need not satisfy the static boundary conditions. The boundary conditions used are as follows :

1. Geometric B.C.

   at $x = 0$; $z = 0$, $y = 0$, $z' = 0$, $y' = 0$,

2. Static B.C.

   at $x = L$; $z'' = 0$, $y'' = 0$, $z''' = 0$, $y''' = 0$

(2.3)

The torsional frequency of straight uniform beam, encastre at one end and free at the other end, allowing for Saint Venant resistance to shear only, is given by,

$$f_n = \frac{n}{4L} \sqrt{\frac{C1}{I_p}} \qquad (2.4)$$

where n = odd integer

$C1$ = stiffness constant

= $G a^3 b / 3$, for rectangular section

L = length the bar

I_p = polar mass moment of inertia

Equation (2.4) is modified by W. Carnegie[10] using an energy method, and has produced a correction factor which allows for the bending of longitudinal fibres during torsion. The frequency equation is given by

$$f_n = \frac{n}{4L} \sqrt{\frac{C1}{I_p}} \sqrt{1 + \left(\frac{n\pi}{L}\right)^2 \frac{C2}{C1}} \qquad (2.5)$$

The corresponding equation for pre-twisted beams of uniform cross section is obtained by replacing C1 by

$$C1 + C3\,(d\alpha / dx)^2 \qquad (2.6)$$

where α = angle of pre-twist

= (x/L)

being the total pre-twist.

W. Carnegie[11] developed total energy expression for a thin blade vibrating in coupled bending - bending - torsion giving fibres due to torsion. Use is made of expressions for PE and KE, equations (2.1) and (2.2), for bending - bending of the blade, derived in (9) and (10).

For calculating expressions for total potential and kinetic energy V_t and T_t for bending - bending torsion of the thin blade, additional terms are

$$V_t = \int_0^L \left[\frac{C\,(\theta')}{2} - \frac{E\,(\theta')(\theta''')}{24} \int_A b^2\, t^3\, db \right] dx \qquad (2.7)$$

$$\equiv \int_0^L \frac{I_{C.G}\,(\theta)^2}{2g}\, dx \qquad (2.8)$$

For aerofoil sections, the centre of twist is away from its C.G, and let the location of shear centre be at distances r_y, r_z in the y and z directions respectively. Then the expression for total K.E becomes

$$T = \int_0^L \left[\frac{(\dot{x} + r_y\,\dot{\theta})^2}{2g} + \frac{(\dot{y} + r_x\,\theta)^2}{2g} + \frac{I_{C.G}\,(\theta)^2}{2g} \right] dx \qquad (2.9)$$

Expression for Lagragian function L is obtained. The Hamilton's Principle is used for getting equations of motion and the boundary conditions. The equations of motion are given by

$$\frac{d^2}{dx^2}\left[E\, I_{yy}\, z'' - E\, I_{zz}\, y'' \right] - \frac{w\,p^2\,z}{g} = \frac{w\,r_y\,p^2\theta}{g} \qquad (2.10)$$

$$\frac{d^2}{dx^2}\left[E\, I_{zz}\, y'' - E\, I_{zz}\, z'' \right] - \frac{w\,p^2 y}{g} = \frac{w\,r_z\,p^2\theta}{g} \qquad (2.11)$$

$$C_1\,\theta''' - C\,\theta'' - \frac{I_{c.f}\,p^2\,\theta}{2} = \frac{w\,r_y\,p^2 z}{g} + \frac{w\,r_z\,p^2 y}{g} \qquad (2.12)$$

A special case of straight blade, of uniform cross section having symmetry about both the principal axes, is considered. The torsional vibration frequencies are calculated which agree closely with those obtained by Rayleigh method and experimental method.

A.P. Duggan and H.A. Slyper[12] used modified finite difference method to equations of motion,[11]

$$C2 \frac{d^4\theta}{dx^4} + \frac{2d\,C2}{dx}\frac{d^3\theta}{dx^3} + \left[\frac{d^2C2}{dx^2} - C3\left(\frac{d\alpha}{dx}\right)^2 - C1\right]\frac{d^2\theta}{dx^2} - \frac{d\,C1}{dx}\cdot\frac{d\theta}{dx} - \frac{d\,C3}{dx}\left(\frac{d\alpha}{dx}\right)^2\frac{d\theta}{dx} = I_p\omega^2\theta \qquad (2.12a)$$

and computed the natural torsional frequencies of vibration of the blade. The blade of uniform cross section is considered. Allowance is made in the analysis for the bending effect of longitudinal fibres during torsion. The results are compared with those obtained experimentally which are within accuracy of 1%. The error is minimum when the beam is divided into elements varying from 30 to 40. The results obtained for rectangular section are comparable with those obtained by Carnegie[11] and the exact solution.

W. Carnegie[13] derived general equation of motion for bending - bending - torsional vibration of pre-twisted cantilever beam. The analysis takes into account the

effect of rotary inertia, shear determination and the effect of bending longitudinal fibre due to torsion. The expression for P.E and K.E are derived. The variation principle is used to derive equations of motion and the boundary conditions. Two important assumptions made

1. Strain energies due to bending, torsion and shear are independent of each other. And,
2. Plane cross-sections remain plane before and after deflection and no warping is produced.

J.S. Rao and W. Carnegie(14) applied Galerkins procedure for finding the natural frequency of straight uniform blade with assymmetric cross-section executing coupled bending - bending - torsional vibrations. The results obtained are comparable with those obtained by a numerical procedure developed by Rao(15) and also with the experimental results obtained by Carnegie & Dawson(16).

W. Carnegie(17) also developed equations of motion of a rotating blade executing small vibrations. In the paper, expression for work done against centrifugal forces is derived. Expressions for total P.E. and K.E. are obtained. Variation principle is used to get characteristic equation of motion for rotating system of general aerofoil section. Rayleigh's method is used to find the fundamental

frequency for the case of uniform blade of symmetrical cross - section.

W. Carnegie[18] in another paper derived differential equations of motion of the same system[17] by including the effect of rotary inertia and the correction factor due to bending of longitudinal fibre during torsion. The author found that the natural frequency depends upon stagger angle ϕ and that the fundamental frequency is unaffected by

1) pre-twist and
2) coupling between bending and torsion.

W. Carnegie and B. Dawson[16] considered the vibration of straight blade of aerofoil cross-section. The theoretical procedure essentially consists of transforming equations (2.10), (2.11) and (2.12) to a set of 10 first order differential equations. Solution of this set is obtained by Runge Kutta step by step numerical integration procedure. For the case of straight beam equations (2.10), (2.11), (2.12) are reduced to

$$E\, I_{xx} \frac{d^4 y}{dx^4} - \frac{w p^2 y}{g} = \frac{w\, z\, p^2 \theta}{g} \tag{2.13}$$

$$E\, I_{yy} \frac{d^4 z}{dx^4} - \frac{w p^2 z}{g} = \frac{w\, p_x\, p^2 \theta}{g} \tag{2.14}$$

$$\frac{C\, d^2 \theta}{dx^2} + \frac{\tau_{c.f}\, p^2 \theta}{g} = - \frac{w r_y\, p^2 z}{g} + \frac{w\, r_z\, p^2 y}{g} \tag{2.15}$$

Solution of the form $z = A e^{\lambda x}$ is assumed. On substitution in the set of ten ordinary differential equations, a secondary quintic equation is obtained in λ^2. Using ten boundary conditions, the set of simultaneous equations are solved for constants of integration.

The results obtained by Runge Kutta method are comparable with the analytical and the experimental results, obtained by Carnegie[11] and are within 0.25% accuracy. G. Esakson and J.G. Eisley[2] discussed both the rotating and non rotating twisted blades for finding the natural frequencies in bending. Both cantilevered and articulated blades are considered. Offset of the support from the axis of rotation has also been included. A relation developed by Lo and Renbarger[20] for the effect of blade root angle ϕ on the natural frequencies is found to provide useful approximation. The analysis makes use of method developed by Targoff[21] which is an extension and adoptation of Holzer-Myklestad method. The Rayleigh Southwell approximation, according to which the PE of the system is found by introducing the work done against the centrifugal force, is used. The relation between the frequencies of rotating and non-rotating blades is given in the form,

$$p_{rn}^2 = p_{Nn}^2 + k_n^2 \Omega^2 \tag{2.16}$$

where

p_{rn} = n^{th} natural frequency for rotating blade.

p_{Nn} = n^{th} natural frequency for non-rotating blade.

k_n = A constant for n^{th} mode and is given by

$$k_n = \frac{\int_1 \left[\int^x \rho \, x \, dx \; \left((y_n)^2 + (z_n)^2 \right) - \rho \, y_n^2 \right] dx}{\int_1 \rho \left(y_n^2 + z_n^2 \right) dx} \tag{2.17}$$

A case study of overhead helicopter propeller blade is presented.

The author concludes that,

1. Fundamental natural frequency of cantilever and non - rotating articulated blade is almost independent of twist if the ratio of major to minor stiffness ratio exceeds three.

2. The second and the third natural frequencies of non - rotating cantilever blade decreases with increasing twist if its maximum stiffness is very large compared to its minimum stiffness.

3. Fundamental natural frequency of a rotating cantilever beam depends primarily on the root blade angle and secondarily on the twist and

4. Offset of the root support increases the frequency.

The theoretical analysis of the twisted turbine blade, both rotating and non rotating, was undertaken by R.C. Diprima and G.H. Handleman[22]. The co-ordinate system used consists of a triad of orthogonal unit rectors, fixed at the root of the blade and another moving triad sliding along the blade length with angular speed equal to the twist per unit length. Effects due to rotary inertia and the shear deformation are excluded. The cantilever of a general cross-section is considered. The beam is assumed to undergo small transverse vibrations. Quantities like deflection, strain and stress are expressed as vector quantities. Equations of motion are obtained by equating the restoring forces to the inertia forces. The same equations of motion are also derived by using variational principle. The solution is obtained by assuming a set of fourth order polynomials for the two deflections and finding the constants of polynomials by trial and error method. The case of a propeller blade is considered for large width to thickness ratio. The results obtained agree closely with the experimental results[23].

So far we have reviewed the work done for a single rotating or non-rotating twisted cantilever blade. Now an account of work done on the packetted turbine blades is given. Only a few papers have appeared in the literature and these concern mainly non-rotating banded blades.

M.A. Prchl[24] has attempted the problem of packetted turbine buckets for the H.P. Section wherein the banded group consists of evenly spaced identical, parallel buckets. For generality blade with hinge flexibility at the root and non uniform cross section is considered. Longitudinal displacements of the blade are assumed negligible with respect to their transverse deflections. Shear deformations and the rotary inertia effects are discarded. His analysis follows the approach used by Smith[26]. The principal axes of moment of inertia of the cross-section are assumed to be parallel and perpendicular to the plane of the wheel on which the banded group is mounted. The centre of twist and its centre of gravity are assumed to coincide. A modified Holzer technique is applied for the two modes of vibration encountered in the bonded blades. These are as follows,

1. Pure tangential mode wherein all displacements are parallel to the plane of rotation and
2. Coupled transverse and torsional mode in which the displacements are perpendicular to the plane of disc and the angular motion is about the longitudinal axis of the blade. The coupling between the transverse and the rotational displacement is due to the band connecting the adjacent blade tips. The use is made

of symmetry and anti symmetry of the banded blades for groups having odd and even number of blades. This helps in reducing the order of frequency determinant to almost half. In his next paper[25], he presented a case study. The paper includes a procedure to determine vibration amplitude and stresses at resonance. Damping is assumed to be small. To determine the stress level, the energy input to the bonded group from a prescribed form of nozzle stimulus is equated to the energy dissipation in damping.

Wilfred Campbell[5] did the experimental investigations for L.P. blades grouped by lashing wires and found the effect of speed of rotation on the natural frequency. An imperial relation between p_{rn} and p_{Nn} was presented

$$p_{rn}^{2} = p_{Nn}^{2} + B N^{2}$$

where B is obtained from the test data for the bonded blades. B for different cases is presented below

(1) $$B = \frac{81}{52} \frac{R}{l} + \frac{61}{52} \tag{2.18}$$

for a bar of uniform section and

(2) $$B = \frac{2R}{l} + \frac{4}{3} \tag{2.19}$$

for a rectangular bar of constant width and tapering to zero in thickness at the free end.

The paper also considers the tunning process in detail. W.C. Heckman in part II of the same paper presented the effect of number of buckets tied together by lacing wires for L.P. section. The plotted results show that the natural frequency increases due to banding.

CHAPTER 3

THEORETICAL ANALYSIS

In the previous chapters, we have stated that the blades in the H.P. section are short, straight, untwisted and invariably banded. The blades have aerofoil cross section and the shear centre of the cross - section is slightly away from the C.G. The principal axes of moment of inertia are not exactly parallel and perpendicular to the plane at the section. In actual practice the band and the blade tip joint may not be perfectly rigid.

As mentioned in the review, the problem of banded H.P. blades was solved by M.A. Prohl[24] by Holzer's Method. He did not consider the effect of centrifugal forces. In the present work, the problem of banded H.P. blades is being analysed for vibrational analysis by using Finite Element Method, incorporating in the effect of centrifugal forces. The object is to collect necessary data for design purposes of banded blades. For this purpose natural frequencies and mode shapes of banded blades are obtained for

a) number of blades in the group being banded
b) band to blade stiffness ratio
c) the rotor speed.

3.1 Assumptions

In the H.P. section, the blades of different stages are fixed to a common rotor and do not transmit vibrations to the rotor[1]. This implies that the vibrations of the blade/banded blades is an independent phenomenon.

The H.P. blades described above, are idealized for carrying out analysis and the assumptions made are mentioned below.

1. The blades banded together have the same configuration and physical dimensions.

2. For the blades, offset of shear centre from the C.G. is very small and are assumed to coincide. The cross section is assumed as rectangular having the same inertial properties as those of original.

3. The staggering angle of the blade root being very small, the principal axes are assumed to be parallel and perpendicular to the plane of wheel or the plane of rotation.

4. The blades are assumed to be rigidly fixed at the root and no hinge flexibility is considered; as such the blades are treated as cantilevers.

5. The rivetted joint between the band and the blade tips is assumed to be perfect. This does not permit relative motion between the band and the blade at the joint.

6. The radius of rotor at the blade root is very large compared to the length of H.P. blade (nearly of the order of 20 : 1). This implies that the banded blades; in a group of 6 to 8, can be considered to be parallel to each other. This facilitates in choosing the nodal co-ordinate axes parallel to each other. Thus no transformation is needed for the superposition of individual elemental matrices.

7. Shear deformation and rotary inertia effects are not considered.

8. The blades being of short length, the longitudinal deflection is assumed to be negligible in comparison to transverse and tangential displacements. This helps in reducing the order of stiffness and inertia matrices for the whole system and

9. External forces considered in the analysis are those which arise from centrifugal action and are of two types. First one is the body force

distributed along the blade length. The second one is force which is produced due to centrifugal force of the mass of the band, concentrated at the blade tip.

3.2 Problem Formulation

Finite Element Method is used to formulate the equations of motion. In the banded blades, the blades and the band are divided into finite elements. A certain co-ordinate system is defined at each nodal point depending upon the mode of vibration under consideration. For each such element, the equation of motion[27] is given by

$$\int_v \rho\, a^T a\, \ddot{U}\, dv + \int_v b^T x_b\, U\, dv = P + \int_s a^T \emptyset\, ds + \int_v a^T X\, dv \tag{3.1}$$

where

- ρ - mass per unit length
- a - a square matrix which relates the local co-ordinates (x and Uy) in terms of nodal displacements U (U2, U6, U8, U12)
- v - volume of the element
- b - a square matrix relating displacement and strain vectors
- P - External concentrated force

xb - a square matrix relating stresses and strains.

$\emptyset$ - surface forces, which are function of local co-ordinates

X - a vector of external body forces expressed in terms of local co-ordinates.

U - vector of nodal displacements.

$\ddot{U}$ - second derivative of nodal displacement U with respect to time.

The equation (3.1) for i^{th} element of the beam becomes

$$M_i \ddot{U}_i + K_i U_i = P_i + \int_{S_i} a_i^T \emptyset_i \, ds + \int_{v_i} a_i^T X_i \, dv \tag{3.2}$$

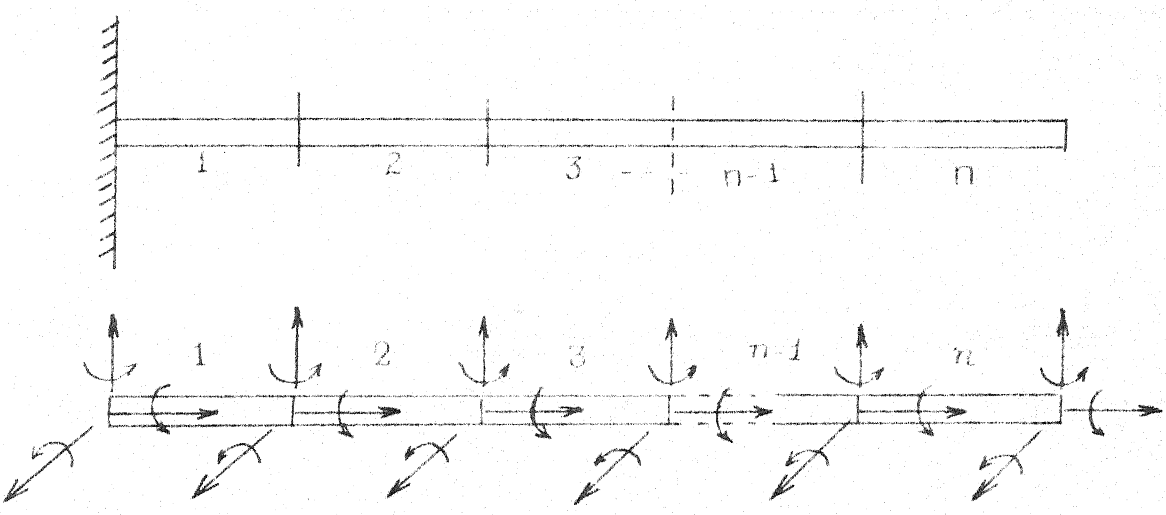
where

$$\left.\begin{aligned} M_i &\triangleq \int_{v_i} a_i^T a_i \, dv && (3.3) \\ K_i &\triangleq \int_{v_i} b_i^T (x_b)_i \, dv && (3.4) \end{aligned}\right\} \text{by definition}$$

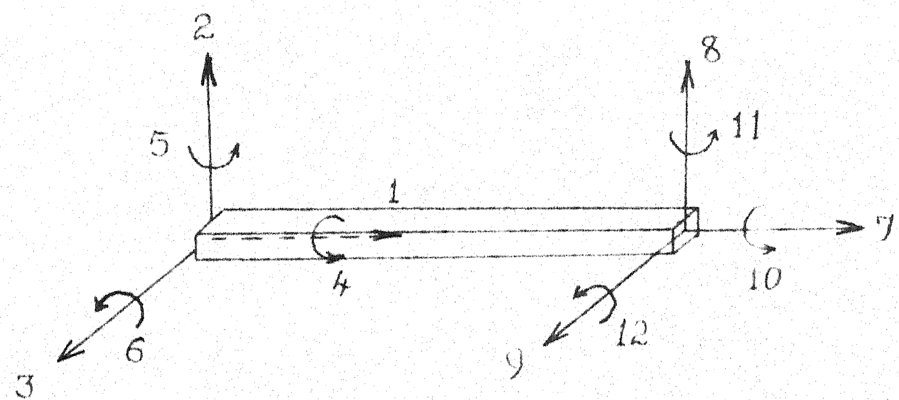
and are called inertia and stiffness matrices. Similarly

$$(P_s)_i = \int_{s_i} a_i^T \emptyset_i \, ds, \quad (P_b)_i = \int_{v_i} a_i^T X_i \, dv$$

are called as generalized surface and body forces respectively. The integrations are carried out for the beam element (Fig. 3.1-b) having six degrees of freedom at each node. These two matrices for the beam element are of the order of 12 x 12 and are given in the Table 1 & 2 respectively.



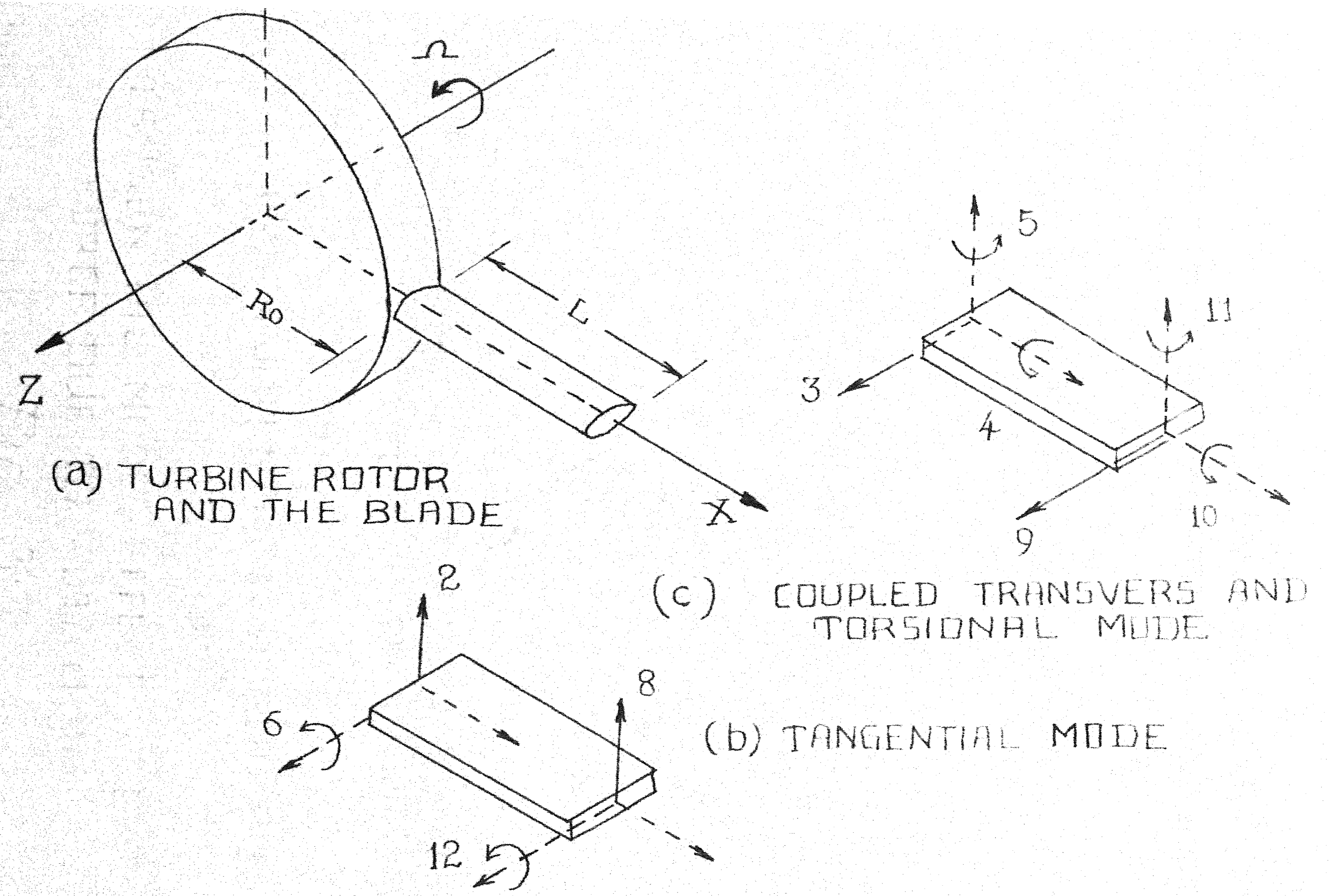
(a) CANTILEVER WITH 'n' ELEMENTS



(b) FINITE ELEMENT

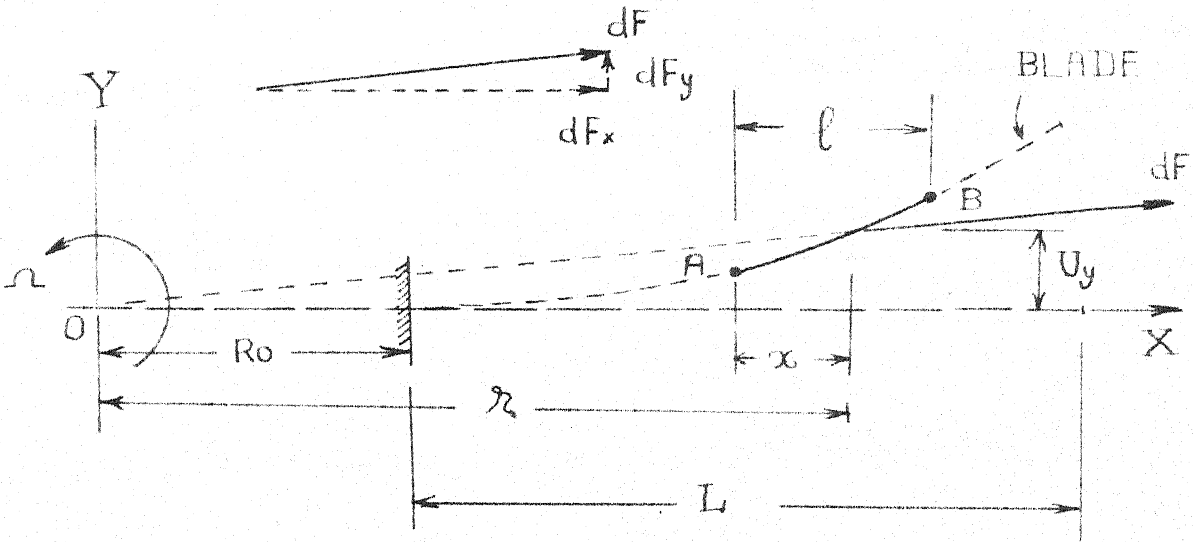
GENERAL CO-ORDINATE SYSTEM
FOR A BEAM

FIG. 3·1

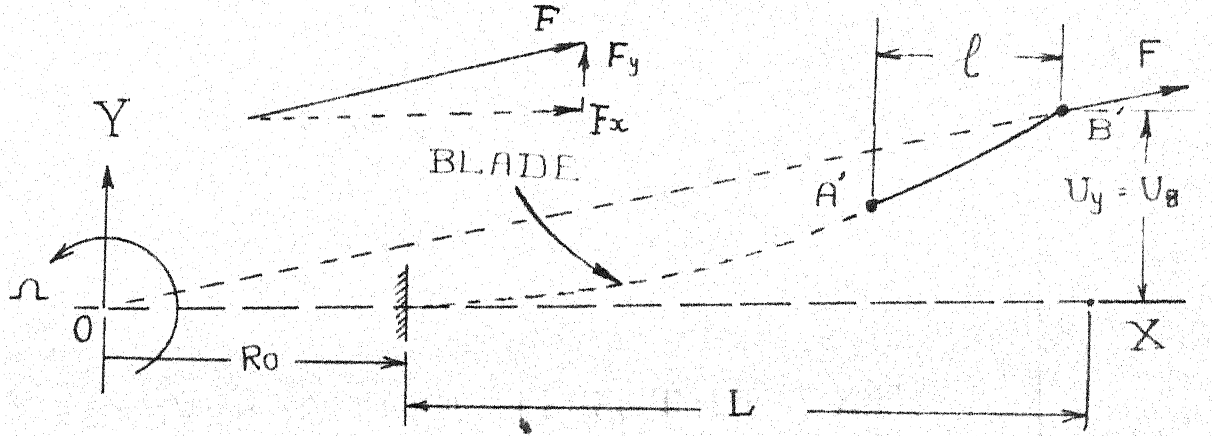


CO-ORDINATE SYSTEM FOR THE TURBINE BLADE

FIG-3·2



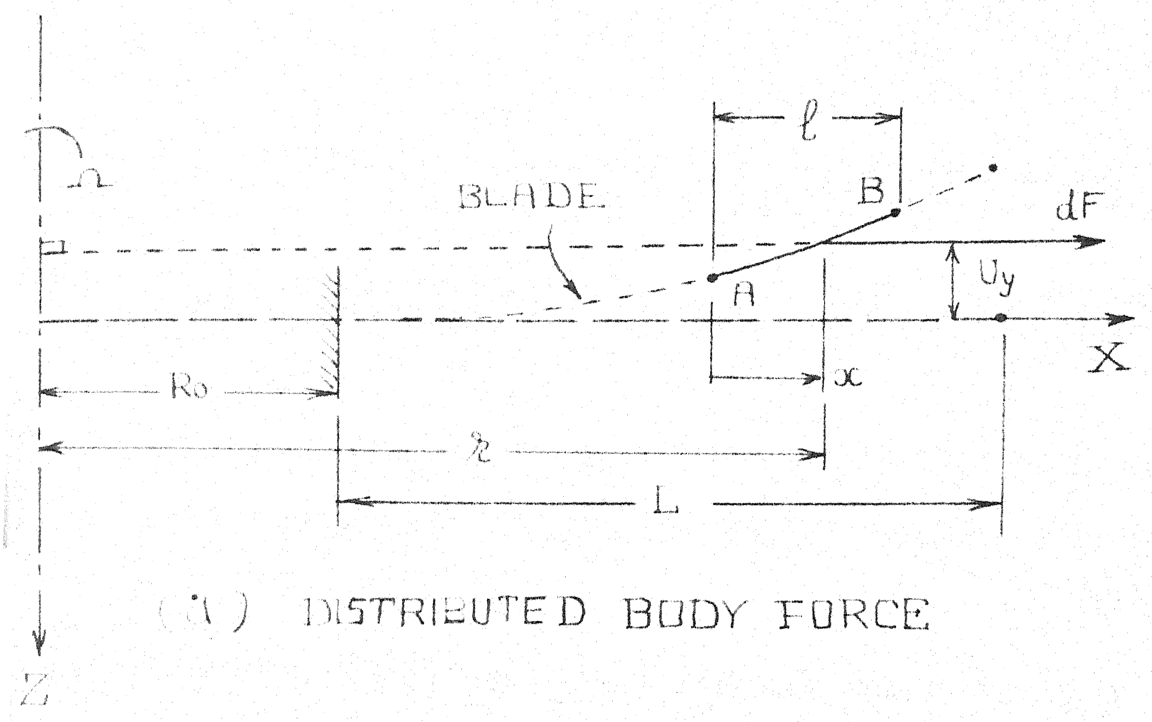
(a) DISTRIBUTED BODY FORCE



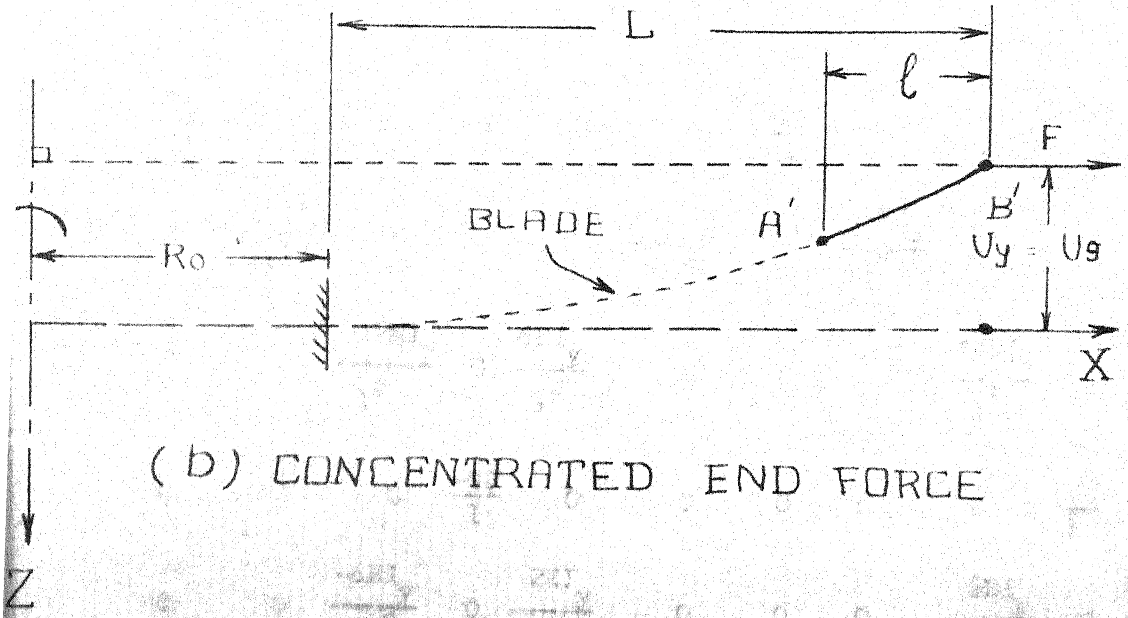
(b) CONCENTRATED END FORCE

TANGENTIAL MODE - EFFECT OF EXT. FORCE DUE TO ROTATION

FIG.- 3·3



(a) DISTRIBUTED BODY FORCE



(b) CONCENTRATED END FORCE

TRANSVERSE MODE-EFFECT OF EXT. FORCE DUE TO ROTATION

FIG-3.4

TABLE - 1

STIFFNESS MATRIX FOR AN ELEMENT OF BEAM WITH 6 DEGREES OF FREEDOM
(Ref. Fig. 3.1-b)

$$
\begin{bmatrix}
\frac{EA}{l} & & & & & & & & & & & \\
0 & \frac{12EI_z}{l^3} & & & & & & & & & & \\
0 & 0 & \frac{12EI_y}{l^3} & & & & & & & & & \\
0 & 0 & 0 & \frac{GJ}{l} & & & & & & & & \text{SYMM.} \\
0 & 0 & \frac{-6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & & & & & & & \\
0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} & & & & & & \\
\frac{-EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{AE}{l} & & & & & \\
0 & \frac{-12EI_z}{l^3} & 0 & 0 & 0 & \frac{-6EI_z}{l^2} & 0 & \frac{12EI_z}{l^3} & & & & \\
0 & 0 & \frac{-12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{12EI_y}{l^3} & & & \\
0 & 0 & 0 & \frac{-GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & & \\
0 & 0 & \frac{-6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & \\
0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} & 0 & \frac{-6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l}
\end{bmatrix}
$$

TABLE - 2

MASS MATRIX FOR AN ELEMENT OF BEAM WITH 6 DEGREES OF FREEDOM
(Ref. Fig. 3.1-c)

$$
\rho A l
\begin{bmatrix}
\frac{1}{3} & & & & & & & & & & & \\
0 & \frac{13}{35} & & & & & & & & & \text{SYMM.} & \\
0 & 0 & \frac{13}{35} & & & & & & & & & \\
0 & 0 & 0 & \frac{J_x}{3A} & & & & & & & & \\
0 & 0 & \frac{-11l}{210} & 0 & \frac{l^2}{105} & & & & & & & \\
0 & \frac{11l}{210} & 0 & 0 & 0 & \frac{l^2}{105} & & & & & & \\
\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & & & & & \\
0 & \frac{9}{70} & 0 & 0 & 0 & \frac{13l}{420} & 0 & \frac{13}{35} & & & & \\
0 & 0 & \frac{9}{70} & 0 & \frac{-13l}{420} & 0 & 0 & 0 & \frac{13}{35} & & & \\
0 & 0 & 0 & \frac{J_x}{6A} & 0 & 0 & 0 & 0 & 0 & \frac{J_x}{3A} & & \\
0 & 0 & \frac{13l}{420} & 0 & \frac{-l^2}{140} & 0 & 0 & 0 & \frac{11l}{210} & 0 & \frac{l^2}{105} & \\
0 & \frac{-13l}{420} & 0 & 0 & 0 & \frac{-l^2}{140} & 0 & \frac{-11l}{210} & 0 & 0 & 0 & \frac{l^2}{105}
\end{bmatrix}
$$

M is called consistent mass matrix. Unlike the mass matrix in lumped mass system, M obtained from equation (3.3) is not a diagonal matrix. Having obtained M_i, K_i for all i's superimpose these in an appropriate manner to obtain mass and stiffness matrices M and K respectively for the whole system. Then the equations of motion take the form

$$M \ddot{U} + K U = P + P_s + P_b \tag{3.5}$$

where U defines all the nodal displacements and M, K, P, P_s and P_b correspond to the coordinate system of U. When only the body forces due to centrifugal action are considered, the equation (3.5) becomes

$$\begin{aligned} M \ddot{U} + K U &= P + P_b \\ &= P + K_b U \end{aligned} \tag{3.6}$$

$$\text{or} \quad M \ddot{U} + (K - K_b) U = P$$

$$\mathbf{M \ddot{U} + \bar{K} U = P}$$

where $\bar{K}$ is the corrected stiffness matrix and $\bar{K} = K - K_b$ is the stiffness correction matrix. For P = 0, let the harmonic solution be of natural frequency p, then equation (3.6) reduces to, ($\bar{K}$ is represented as K without any loss of generality)

$$- p^2 M U + K U = 0$$

Premultiplying by K^{-1} throughout and rearranging, we get

$$K^{-1} M U = \frac{1}{p^2} U$$

or

$$D\ U = \lambda\ U \tag{3.7}$$

where

$$D = K^{-1} M$$

$$\lambda = \frac{1}{p^2} \tag{3.8}$$

Thus the problem of the vibration of the system reduces to an eigen value problem, equation (3.7). Matrix D whose eigen values are to be found is called Dynamical Matrix and is given by the equation (3.7). The eigen values give the characteristic values of the system and these give its natural frequencies. The eigen vector U corresponding to eigen vc gives the mode shape of vibration for the natural frequency given by λ.

Fig. 3.2-a shows the configuration of the turbine blade mounted on the turbine rotor. The orthogonal co-ordinate system (X, Y, Z) are selected as shown in the figure. Two modes of vibrations are considered. Fig. 3.2-b shows the co-ordinate system chosen for the tangential mode of vibrations wherein the displacements (U_2, U_6, U_8, U_{12}) occur in the X - Y plane. Fig. 3.2-c shows the co-ordinate system for the coupled transverse and the torsional vibrations wherein the displacements (U_3, U_4, U_5, U_9, U_{10}, U_{11}) occur in the X - Z plane and the angular rotations about the individual blade axis.

3.3 Derivation of Stiffness Correction Matrix

In the last section, we have presented how the mass and stiffness matrices for the system are found from equations (3.3) and (3.4). It is also mentioned that depending upon the external forces, equation (3.5) gets modified. In this section we will consider the effect of external forces in two modes of vibrations namely tangential and coupled transverse - torsional modes. Equation (3.5) is reproduced here as

$$M \ddot{U} + K U = P + \int_V a^T X \, dv$$

$$= P + P_b \qquad (3.9)$$

Effects due to concentrated force P and the distributed force X are considered seperately and these are found in both the modes of vibration. Thus the four correction matrices derived are as follows,

a. Tangential mode

K_1 - takes into account the effect of distributed body force of the blade element due to centrifugal action.

K_2 - takes into account the effect of rotating mass of the band connected at the end of the blade.

b. Coupled transverse and torsional mode

K_3 - takes into account the distributed body force of the blade element due to rotation,

K_4 - considers the effect of rotating mass of the band connected at the end of the blade.

These are derived below.

3.3.1 Expression for K_1

Fig. ~~7.3-a~~ ~~1.20~~ 3.39 shows an element of the blade mounted on the rotor and executing tangential mode of vibration in the X-Y plane. The rotor rotates at a constant angular velocity of Ω. The nodal co-ordinates U_2, U_6, U_8, U_{12} have already been shown in figure (~~2b~~ 1.2b). The inner end of the element is at a distance R from the axis of rotor.

For the deflection curve corresponding to tangential vibrations the distribution of body force X is found below :

Let

dF = Force in the radial direction OC due to a small element dx situated at a distance x from the inner end **A**.

dF_x = Component of the centrifugal force dF along X axis

dF_y = component of the centrifugal force dF along Y axis.

U_y = Tangential displacement in the Y direction of the element situated at distance x.

m = mass per unit length $= \rho . A$.

A = Cross-sectional area of the blade (which is constant in this case).

When the rotating blade is not vibrating, the body force acts along the blade axis x but when the blade starts vibrating, the transverse displacement U_y gives rise to component dF_y in the Y direction. This vertical component of centrifugal force affects the vibrational phenomena by introducing additional restoring forces which is a linear function of displacement U_y. For an element at a distance r and having a transverse displacement U_y,

the radial distance OC $= \sqrt{r^2 + U_y^2}$

Hence the centrifugal force dF, its components dF_x and dF_y and the body couple are

$$dF = m \sqrt{r^2 + U_y^2}\ \Omega^2$$

$$dF_x = m \sqrt{r^2 + U_y^2}\ \Omega^2 \times \frac{r}{\sqrt{\left[r^2 + U_y^2\right]}} = m\, r\, \Omega^2$$

$$dF_y = m \sqrt{r^2 + U_y^2}\ \Omega^2 \ \frac{r}{\sqrt{\left[r^2 + U_y^2\right]}} = m\, U_y \Omega^2$$

$$dM_{xy} = 0$$

The external body force $X_1 . \triangleq \begin{Bmatrix} d\,F_x \\ d\,F_y \\ d\,M_{xy} \end{Bmatrix}$ is

$$X_1 = \begin{Bmatrix} M\,r\Omega^2 \\ M\,U_y\Omega^2 \\ 0 \end{Bmatrix} \qquad (3.10)$$

U_y is expressed in nodal displacements (U_2, U_6, U_8, U_{12})

where,

$$U_y = (1 - 3\xi^2 + 3\xi^3)\,U_2 + (\xi - 2\xi^2 + \xi^3)\,l\,U_6$$

$$+ (3\xi^2 - 2\xi^3)\,U_8 + (-\xi^2 + \xi^3)\,l\,U_{12} \qquad (3.11)$$

where l = length of the finite element AB

$\xi = x/l$, x being distance measured from A.

When the expression (3.10) for X_1 is used in equation (3.9), it becomes,

$$M\ddot{U} + K\,U = \int_v a^T X\,dv = F_1^*, \text{ say }, \qquad (3.13)$$

where the integration is carried over the volume of the blade. When the value of U_y from expression (3.11) is substituted in the R.H.S. of equation (3.12), F_1^* is found to consists of two parts, namely

F_1 = component of F_1^* which is a function of nodal displacements, U_2, U_6, U_8, U_{12}.

F'_1 = a constant vector

F_1^* is then written as,

$$F_1^* = K_1 U + F'_1 \tag{3.12}$$

where K_1 = square matrix.

Replacing F_1^* in equation (3.12) by the R.H.S. of (3.12), we get

$$M \ddot{U} + K U = K_1 U + F'_1$$

or

$$M \ddot{U} + \bar{\bar{K}}_1 U = F_1'$$

where

$\bar{\bar{K}}_1$ = $\left[K - K_1\right]$ and is called the corrected stiffness matrix

K = original stiffness matrix for stationary element

K_1 = stiffness correction matrix.

The correction stiffness matrix K_1 is given in table 3a.

3.3.2 Expression for K_2

Fig. 3.3-b shows an element at the tip of a blade. The force F, due to centrifugal effect of rotating mass M of the band, is shown to act at the tip B. We find below the expression for K_2 due to the body force of the band which acts concentrated on the blade end. Hence P_s and P_b are zero.

TABLE - 3

a) STIFFNESS CORRECTION MATRIX K_1

$$K_1 = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

Where,

$$A = \begin{vmatrix} l^2\left(\frac{1}{2}\,\frac{R}{l} + \frac{13}{35}\,\frac{1}{l} + \frac{13}{70}\right) & l^3\left(\frac{R}{10l} + \frac{11}{210l} + \frac{3}{70}\right) \\ l^3\left(+\frac{R}{10l} + \frac{11}{210l} + \frac{1}{105}\right) & l^4\left(\frac{1}{210} + \frac{1}{105l}\right) \end{vmatrix}$$

$$B = \begin{vmatrix} l^2\left(\frac{R}{2l} + \frac{9}{70l} + \frac{11}{35}\right) & l^3\left(-\frac{R}{10l} - \frac{13}{420l} - \frac{2}{35}\right) \\ l^3\left(\frac{R}{10l} + \frac{13}{420l} + \frac{31}{420}\right) & l^4\left(-\frac{R}{60l} - \frac{1}{84} - \frac{1}{140l}\right) \end{vmatrix}$$

$$C = \begin{vmatrix} l^2\left(-\frac{R}{2l} - \frac{13}{70} + \frac{9}{70l}\right) & l^3\left(-\frac{R}{10l} - \frac{3}{70} + \frac{13}{420l}\right) \\ l^3\left(\frac{R}{10l} + \frac{11}{420} - \frac{13}{420l}\right) & l^4\left(-\frac{R}{60l} + \frac{1}{210} - \frac{1}{140l}\right) \end{vmatrix}$$

$$D = \begin{vmatrix} l^2\left(-\frac{R}{2l} + \frac{13}{35l} - \frac{11}{35}\right) & l^3\left(\frac{R}{10l} + \frac{2}{35} - \frac{11}{210l}\right) \\ l^3\left(-\frac{R}{10l} - \frac{23}{210} - \frac{11}{210l}\right) & l^4\left(\frac{1}{210} + \frac{1}{105l}\right) \end{vmatrix}$$

b) STIFFNESS CORRECTION MATRIX K_2

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & M^2 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

In this case, we have,

$$F = M \sqrt{(R + 1)^2 + U_8{}^2}\, \Omega^2$$

$$\begin{matrix} F_x & = \\ F_y & = \\ M_{xy} & = \end{matrix} \begin{Bmatrix} M \quad (R + 1)\,\Omega^2 \\ M \quad U_8 \Omega^2 \\ 0 \end{Bmatrix}$$

The external force vector X_2 is given by

$$X_2 = \begin{Bmatrix} M\,(R + 1)\,\Omega^2 \\ M\,U_8 \Omega^2 \\ 0 \end{Bmatrix} \tag{3.14}$$

Since the external force X_2 is acting at the node itself, no transformation is necessary to get the vector X_2 in terms of nodal coordinates.

Seperating X_2 in two parts, we get

$$X_2 = Q_2 + Q_2'$$

where

Q_2 = a linear function of displacements U

Q_2' = a constant vector

Substituting the values of X_2 in equation (3.9), it becomes

$$M\ddot{U} + K\,U = K_2\,U + Q_2'$$

where K_2 is the correction stiffness matrix

Simplifying

$$M\ddot{U} + (K - K_2)\,U = Q_2$$

or $$M\ddot{U} + \bar{K}_2\,U = Q_2{}'$$

$\bar{K}_2$ is called the corrected stiffness matrix for the case being discussed. Elements of the matrix K_2 are shown in table 3b.

3.3.3 Expression for K_3

Figure 4-a shows the configuration of the forces for the case of combined transverse - torsional mode. In this dase, the body force due to centrifugal action is parallel to the X axis, and the transverse component of this force is always zero. Hence, for this case,

$$dF = d\,F_x = m\,r\,\Omega^2\,dx$$

$$dF_y = 0$$

$$d\,M_{xy} = 0$$

$$X_3 = \begin{Bmatrix} M\,(R + x)\,\Omega^2 \\ 0 \\ 0 \end{Bmatrix} \qquad (3.15)$$

Substituting this value of X_3 in (3.9), we get the equation of the type $M\ddot{U} + \bar{K}_3\,U = X_3{}'$, as before

TABLE - 4

a) STIFFNESS CORRECTION MATRIX K_3

$$
\begin{matrix}
l^2\left(\frac{R}{21}+\frac{13}{70}\right) & l^3\left(\frac{R}{10l}+\frac{3}{70}\right) & l^2\left(\frac{R}{21}+\frac{11}{35}\right) & l^3\left(-\frac{R}{10l}-\frac{2}{35}\right) \\
l^3\left(-\frac{R}{10l}+\frac{1}{105}\right) & l^4\left(\frac{1}{210}\right) & l^3\left(\frac{R}{10l}+\frac{31}{420}\right) & l^4\left(-\frac{R}{60l}-\frac{1}{84}\right) \\
l^2\left(-\frac{R}{21}-\frac{13}{70}\right) & l^3\left(-\frac{R}{l}-\frac{3}{70}\right) & l^2\left(-\frac{R}{21}-\frac{11}{35}\right) & l^3\left(\frac{R}{10l}+\frac{2}{35}\right) \\
l^3\left(\frac{R}{10l}+\frac{11}{420}\right) & l^4\left(\frac{R}{60l}+\frac{1}{210}\right) & l^3\left(-\frac{R}{10l}-\frac{23}{210}\right) & l^4\left(\frac{1}{210}\right)
\end{matrix}
$$

b) STIFFNESS CORRECTION MATRIX K_4

$$
\begin{matrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{matrix}
$$

NOTE: Matrices are related to coordinates, U_3, U_5, U_9, U_{11}

where

$$\bar{K}_3 = K - K_3$$

Value of K_3 is given in the table 4a.

3.3.4 Expression for K_4

Fig. 3.4-b shows the force F acting at the end B'. Since the forces are parallel to X axis, $F_y = 0$ and

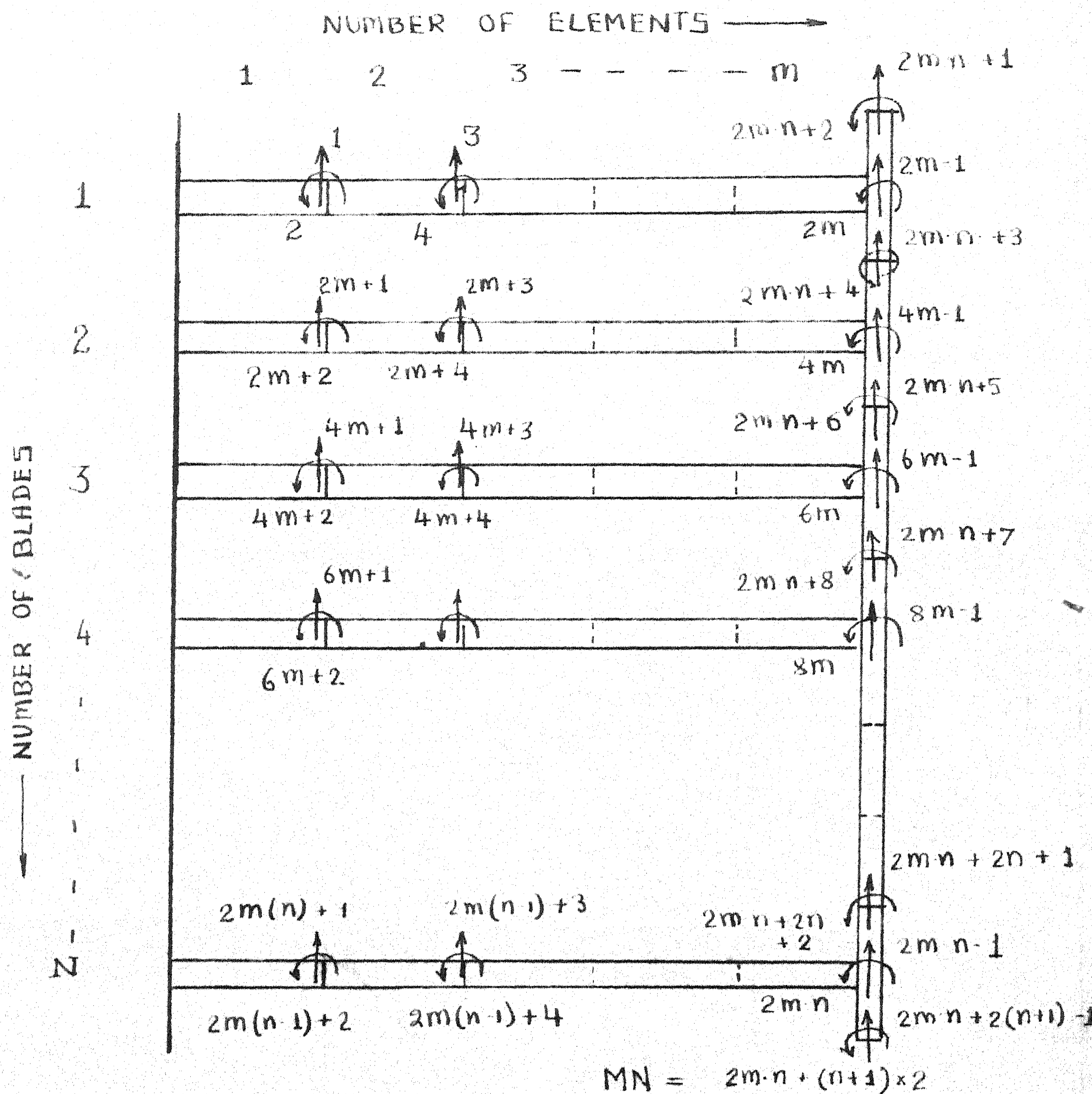
$$X_4 = Q_4 + Q'_4 = 0 + Q'_4$$

$$= K_4 U + Q'_4$$

Here K_4 is a null matrix. So when substituted in equation (3.9) it does not affect K and is given in table 4b.

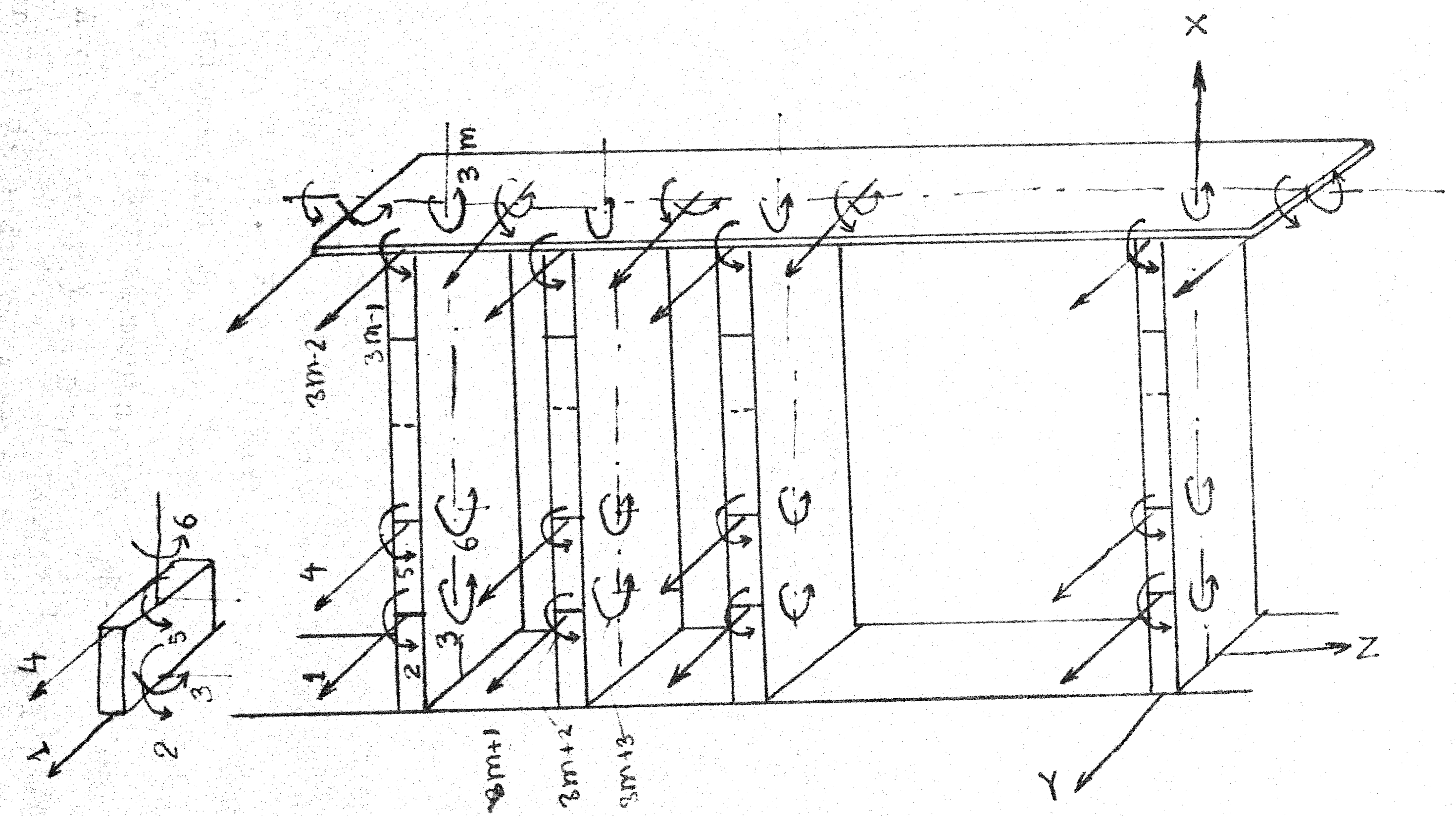
3.4 Dynamical Matrix Of The Packetted Group Of Turbine Blades

The packetted group of blades is an assemblage of individual blades and an overhung band element, connected at the free ends of the cantilever blades. The blades and the band element are divided into several segments. Nodal co-ordinate system is defined depending upon the mode of vibration. The stiffness and mass matrices for an element have been obtained in Section 3.2. Correction stiffness matrices have also been derived in Section 3.3. Mass and corrected stiffness matrices M and K for the sequence



CO-ORDINATE SYSTEM FOR TANGENTIAL MODE

FIG 3.5



CO-ORDINATE SYSTM FOR COUPLED TRANSVERSE - TORSIONAL MODE

FIG. 3·6

of coordinates system are obtained by super posing $\overline{M}_i$ and $\overline{K}_i$ of the elements subject to certain mode of vibration.

3.4.1 Tangential Mode of Vibration.

Figure 3.5 shows a packetted group of n blades. Each blade and band are divided in m and 2 elements respectively. An additional node is defined at the end of each overhanging band. The total number of nodes is thus given as

$$m \cdot n + (n + 1). \tag{3.16}$$

It has been presented that for tangential mode two co-ordinates are necessary for each node. Hence total number of co-ordinates for the system is,

$$2\, m.n + 2\,(n + 1). \tag{3.17}$$

Tables 5 and 5a show the mass and the stiffness matrices M_i, K_i for the blade and the band elements respectively, which are obtained from the tables 1 and 2 by deleting those rows and columns for which the coordinates are zero. Having found M_i and K_i for all elements, superposition is done in the following manner. For the first element of the blade, first two columns and rows are zero because of the boundary conditions. For other elements, the superposition is done by adding A of M_i (or K_i) to D of M_{i-1} (or K_{i-1}) similarly D of M_i (or K_i) is added to A of M_{i+1} (or K_{i+1}).

Here A, B, C, D are the sub-matrices of M_i (or K_i) as shown in the Table 5. This superposition is carried out for all the elements of the blade. The band has 2 n elements and (2n + 1) nodes of which alternate node is common to the node of the blade. Note that this corresponds to the tip of the blade.

For each element of the band, M_i (or K_i) has already been calculated. These are superposed so that A and B of M_i (or K_i) are added in the following manner,

a) for the starting node of the band, add A corresponding to 2mn+1, 2mn+2 rows and column

b) add D of first element of band to D of M_m(or K_m)

c) for any element (mn + k) of the band, two cases arise

1) k is odd, then add D of M_{mn+k} (or K_{mn+k}) to D of $M_{(\frac{k+1}{2})}$ (or $K_{m(\frac{k+1}{2})}$) and A of M_{mn+k} (or K_{mn+k}) to D of M_{mn+k-1} (or K_{mn+k-1})

2) k is even, then add A of M_{mn+k} (or K_{mn+k}) to D of $M_{m(\frac{k-1}{2})}$ (or $K_{m(\frac{k-1}{2})}$) and D of M_{mn+k} or (K_{mn+k}) to A of M_{mn+k+1} (or K_{mn+k+1}).

Note no change in values of B's and C's of M_i (or K_i) of blade and band takes place during superposition to get M and K.

TABLE - 5a

TANGENTIAL MODE

BLADE PROPERTIES

STIFFNESS MATRIX

$$\begin{bmatrix} \frac{12EI_z}{l^3} & \frac{6EI_z}{l^2} & \frac{-12EI_z}{l^3} & \frac{6EI_z}{l^2} \\ \frac{6EI_z}{l^2} & \frac{4EI_z}{l} & \frac{-6EI_z}{l^2} & \frac{2EI_z}{l} \\ \frac{-12EI_z}{l^3} & \frac{-6EI_z}{l^2} & \frac{12EI_z}{l^3} & \frac{-6EI_z}{l^2} \\ \frac{6EI_z}{l^2} & \frac{2EI_z}{l} & \frac{-6EI_z}{l^2} & \frac{4EI_z}{l} \end{bmatrix}$$

INERTIA MATRIX

$$Al \begin{bmatrix} \frac{13}{35} & \frac{11l}{210} & \frac{9}{70} & \frac{-13l}{420} \\ \frac{11l}{210} & \frac{l^2}{105} & \frac{13l}{420} & \frac{-l^2}{140} \\ \frac{9}{70} & \frac{13l}{420} & \frac{13}{25} & \frac{-11l}{210} \\ \frac{-13l}{420} & \frac{-l^2}{140} & \frac{-11l}{210} & \frac{l^2}{105} \end{bmatrix}$$

Note:

The matrices are partitioned into four (2 x 2) sub-matrices, which facilitates the superposition process. The partitioned matrix is shown below as,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

TABLE - 5b

TANGENTIAL MODE

BAND PROPERTIES

STIFFNESS MATRIX

$$\begin{bmatrix} \frac{AE}{l} & 0 & \frac{-AE}{l} & 0 \\ 0 & \frac{4EI_z}{l} & 0 & \frac{2EI_z}{l} \\ \frac{-AE}{l} & 0 & \frac{AE}{l} & 0 \\ 0 & \frac{2EI_z}{l} & 0 & \frac{4EI_z}{l} \end{bmatrix}$$

INERTIA MATRIX

$$\rho Al \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{l^2}{105} & 0 & \frac{-l^2}{140} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{-l^2}{140} & 0 & \frac{l^2}{105} \end{bmatrix}$$

3.4.2 Coupled Transverse - Torsional Mode

Figure 3.6 shows the co-ordinate system for the packetted turbine blades when vibrating in the combined transverse torsional mode. The number of node points is still given by equation 3.16. The number of degrees of freedom in this case is given as,

$$3\ m.n + 3\ (n + 1) \tag{3.18}$$

Table 6a and 6b show the elemental stiffness and mass matrices for both the blade and the band which are derived from table 1 & 2, by properly deleting the columns and rows for which the co-ordinates are zero.

Superposition of these matrices M_i & K_i to give M & K for the whole system, follows the same procedure described in the last section.

TABLE - 6a

COUPLED TRANSVERSE - TORSIONAL MODE
BLADE PROPERTIES

Stiffness Matrix

$$
\begin{matrix}
\frac{12EI_y}{l} & \frac{-6EI_y}{l^2} & 0 & \frac{-12EI_y}{l^3} & \frac{-6EI_y}{l^2} & 0 \\[2ex]
\frac{-6EI_y}{l^2} & \frac{4EI_y}{l} & 0 & \frac{6EI_y}{l^2} & \frac{2EI_y}{l} & 0 \\[2ex]
0 & 0 & \frac{GJ}{l} & 0 & 0 & \frac{-GJ}{l} \\[2ex]
\frac{-12EI_y}{l^3} & \frac{6EI_y}{l^2} & 0 & \frac{12EI_y}{l^3} & \frac{6EI_y}{l^2} & 0 \\[2ex]
\frac{-6EI_y}{l^2} & \frac{2EI_y}{l} & 0 & \frac{6EI_y}{l^2} & \frac{4EI_y}{l} & 0 \\[2ex]
0 & 0 & \frac{-GJ}{l} & 0 & 0 & \frac{GJ}{l}
\end{matrix}
$$

Inertia Matrix

A1

$$
\begin{matrix}
\frac{13}{35} & \frac{-11l}{210} & 0 & \frac{9}{70} & \frac{13l}{420} & 0 \\[2ex]
\frac{-11l}{210} & \frac{l^2}{105} & 0 & \frac{-13l}{420} & \frac{-l^2}{140} & 0 \\[2ex]
0 & 0 & \frac{J_x}{3A} & 0 & 0 & \frac{J_x}{6A} \\[2ex]
\frac{9}{70} & \frac{-13l}{420} & 0 & \frac{13}{35} & \frac{11l}{210} & 0 \\[2ex]
\frac{13l}{420} & \frac{-l^2}{140} & 0 & \frac{11l}{210} & \frac{l^2}{105} & 0 \\[2ex]
0 & 0 & \frac{J_x}{6A} & 0 & 0 & \frac{J_x}{3A}
\end{matrix}
$$

TABLE – 6b

COUPLED TRANSVERSE – TORSIONAL MODE
BAND PROPERTIES

STIFFNESS MATRIX

$$\begin{bmatrix} \frac{12EI_x}{l^3} & \frac{6EI_x}{l^2} & 0 & \frac{-12EI_x}{l^3} & \frac{6EI_x}{l^2} & 0 \\ \frac{6EI_x}{l^2} & \frac{4EI_x}{l} & 0 & \frac{-6EI_x}{l^2} & \frac{2EI_x}{l} & 0 \\ 0 & 0 & \frac{G\,J_y}{l} & 0 & 0 & -\frac{G\,J_y}{l} \\ \frac{-12EI_x}{l^3} & \frac{-6EI_x}{l^2} & 0 & \frac{12EI_x}{l^3} & \frac{-6EI_x}{l^2} & 0 \\ \frac{6EI_x}{l^2} & \frac{2EI_x}{l} & 0 & \frac{-6EI_x}{l^2} & \frac{4EI_x}{l} & 0 \\ 0 & 0 & \frac{-G\,J_y}{l} & 0 & 0 & \frac{G\,J_y}{l} \end{bmatrix}$$

INERTIA MATRIX

$$A l \begin{bmatrix} \frac{13}{35} & \frac{11l}{210} & 0 & \frac{9}{70} & \frac{-13l}{420} & 0 \\ \frac{11l}{210} & \frac{l^2}{105} & 0 & \frac{13l}{420} & \frac{-l^2}{140} & 0 \\ 0 & 0 & \frac{J_y}{3A} & 0 & 0 & \frac{J_y}{6A} \\ \frac{9}{70} & \frac{13l}{420} & 0 & \frac{13}{35} & \frac{-11l}{210} & 0 \\ \frac{-13l}{420} & \frac{-l^2}{140} & 0 & \frac{-11l}{210} & \frac{l^2}{105} & 0 \\ 0 & 0 & \frac{J_y}{6A} & 0 & 0 & \frac{J_y}{3A} \end{bmatrix}$$

CHAPTER 4

RESULTS AND CONCLUSIONS

In the last chapter we have formulated the Dynamical Matrix for the two modes of vibrations. Respective correction matrices are incorporated to account for the effect of rotation of the turbine rotor. In this chapter, the eigen value problem formulated in the previous chapter is numerically solved for both the modes of vibrations.

4.1 Order of Dynamical Matrix

Each blade in the banded group is divided into 5 and 4 elements for tangential and coupled Transverse - Torsional modes respectively. In both the cases the band spanning any two blade is divided into two elements. Calculation of natural frequencies for the banded group is done for blades numbering one to six. The order MN of Matrix for N number of blades for two modes of vibration is given by, (Refer. Fig. 3.1, 3.2).

a) Tangential Mode

$$MN = 12\,(N) + 2 \qquad \text{for } n \geqslant 2$$
$$= 10 \qquad \text{for } n = 1$$

b) Coupled Transverse - Torsional Mode

$$MN = 15\,(N) + 3 \qquad \text{for } n \geqslant 2$$
$$= 12 \qquad \text{for } n = 1$$

For one blade, the values of natural frequencies obtained are shown in Table (7). The table also shows the theoretically obtained values of natural frequencies when the blade is considered as a continuous system. The error involved between the two results is in the range of 0.11% to 5.75% for fundamental frequency to natural frequency.

4.2 Specifications of the Banded Group

The following data is used to calculate the natural frequency of the banded group; used in the H.P. Section of the 130 MW ~~Stem~~ Steam turbine, manufactured by the Heavy Electricals (India) Limited, Bhopal.

1.	Length of the blade	: 0.88 inches
2.	Cross Sectional area of the blade	: 0.325 in^2
3.	Moment of Inertia	
	-about zz axis	: 0.0137 in^4
	-about yy axis	: 0.0089 in^4
4.	Width of band D	: 1.03 inch
5.	Thickness of band B	: 0.105 inch
6.	Blade pitch, S	: 0.76 inch
7.	Young's Modulus for both blade and band	: 30×10^6 lb/in^2
8.	Modulus of reaiding for blade and the band	: 10×10^6 lb/in^2

9. Specific density for the material of band and the blade. : 0.28 lb/in^3

10. Overhung of the band : 0.38 inch

11. Rotor radius at the blade root: 17.5 inch

12. Rotor speed : 3000 rpm

4.3 Method for calculation of Eigen Values and Eigen Vectors

Matrix iteration method is used for the solution of the eigen value problem (3.7). The iteration is started by a trial mode $\{1, 1, \ldots 1\}$ which is premultiplied by D. The resulting column matrix is then normalized by reducing the largest numerical element to unity i.e. for

$$DU_i = \lambda_i \; U_{i+1} \tag{4.1}$$

where $U_o = \left\{1, 1, \ldots\ldots 1\right\}$ and U_{i+1} is normalized for all i.

i = 20 is sufficient for the convergence of the normalized vector. The normalizing quantity gives the largest eigen value for D, which is proportional to $1/p^2$. This gives the corresponding minimum frequency, p.

For obtaining the next dominant eigen value of D, Wielandt's Deflation Procedure[28] is used which reduces the matrix order by one and eliminates eigen value

corresponding to $\lambda\ max^m$. The iterative procedure gives the next dominant eigen value, which in turn gives next lowest natural frequency. Repetition of this method gives all the eigen values in order. Note eigen vectors are not obtained by this method.

Since only a few eigen vectors are desired another iterative procedure is used wherein the order of matrix is not reduced. Sweeping procedure[29] is used to supress the unwanted eigen values and the corresponding eigen vectors.

4.4 Numerical Computation

Computer programs are made for the evaluation of natural frequencies for tangential mode and coupled transverse - torsional mode based on the formulation given in Section 4.3. Fortran IV language is used for the programs which are run on IBM 7044 - 1401 Digital Computer. The details of the Computer Program are given in Appendix D.

4.5 Results Obtained

First six natural frequencies for both the modes are calculated for the packetted turbine blades, N = 2 to 6, for blade data given in Section 4.2 and are tabulated in Appendix B for ζ = 0.005 to 0.006. In Appendix A data is tabulated for effect of centrifugal force of rotating

packetted blades for N = 2 and 6 and ξ = 0.02 (0.0231) for both the modes of vibrations and are plotted in graphs 4.1 (a) and 4.1 (b). For different banded groups frequencies for fundamental to sixth modes are plotted, a) against N for parameter ξ and b) against ξ for parameter N. These are given in graphs 4.4.1 to 4.4.6 and 4.3.1 to 4.3.6, for tangential and coupled transverse - torsional vibration modes respectively.

4.6 Discussion of the Results

Mode shapes for first two natural frequencies for N = 2 to 6 are plotted in graphs 4.2.1 to 4.2.2.

The results obtained for effect of single blade show that Ω has no effect on the natural frequencies. This is quite obvious in the case of very short blades. Results for the banded blades, rotor graphs 4.1 show clearly no effect of Ω on natural frequencies implying the p's for banding of short blades are not affected by centrifugal action. Since cases of I.P. and L.P. blades are not considered no categorial statement on effect of Ω on p's is given but it is expected that for I.P. and L.P. there should be prominent effect.

Results show that there is marked fall in natural frequencies p of banded blades from unbanded single blade.

The graphs 4.4.1 to 4.4.6 are for tangential mode of vibration of banded blades; for p_1 bending has no effect which is clear from the mode shapes, refer graph 4.4.1 as mode of $N > 2$ is repetition of modes for $N = 2$ where as for p_i, $i = 2, 3 \ldots$, there is a decreasing effect due to N for all ξ. These graphs also show that ξ has marginally increasing effect on all p_i, $i = 2, 3, 4$ for banded blades $N = 2, 3, 4 \ldots$

The graphs 4.3.1 to 4.3.6 show the results for coupled transverse - torsional vibrations.

Graph 4.3.1 shows that the fundamental frequency increases by very small amount as the number of blades is increased from $N = 2$ to 6. This is clear from Fig. 4.4.2, for mode shape, which shows that by adding more number of blades it puts constraints on the horizontal deflection of the overhanging band tips. The graphs 4.3.2 to 4.3.6 show that for p_i, $i = 2, 3, \ldots 6$, there is a decreasing effect for the banded blades $N = 2, 3, \ldots 6$. These graphs also show that ξ has marginally decreasing effect on p_i, $i = 2, 3, \ldots$.

From the graphs of p against ξ we note that the range of frequency change is more for the case of changing the number of blades N than that of ξ.

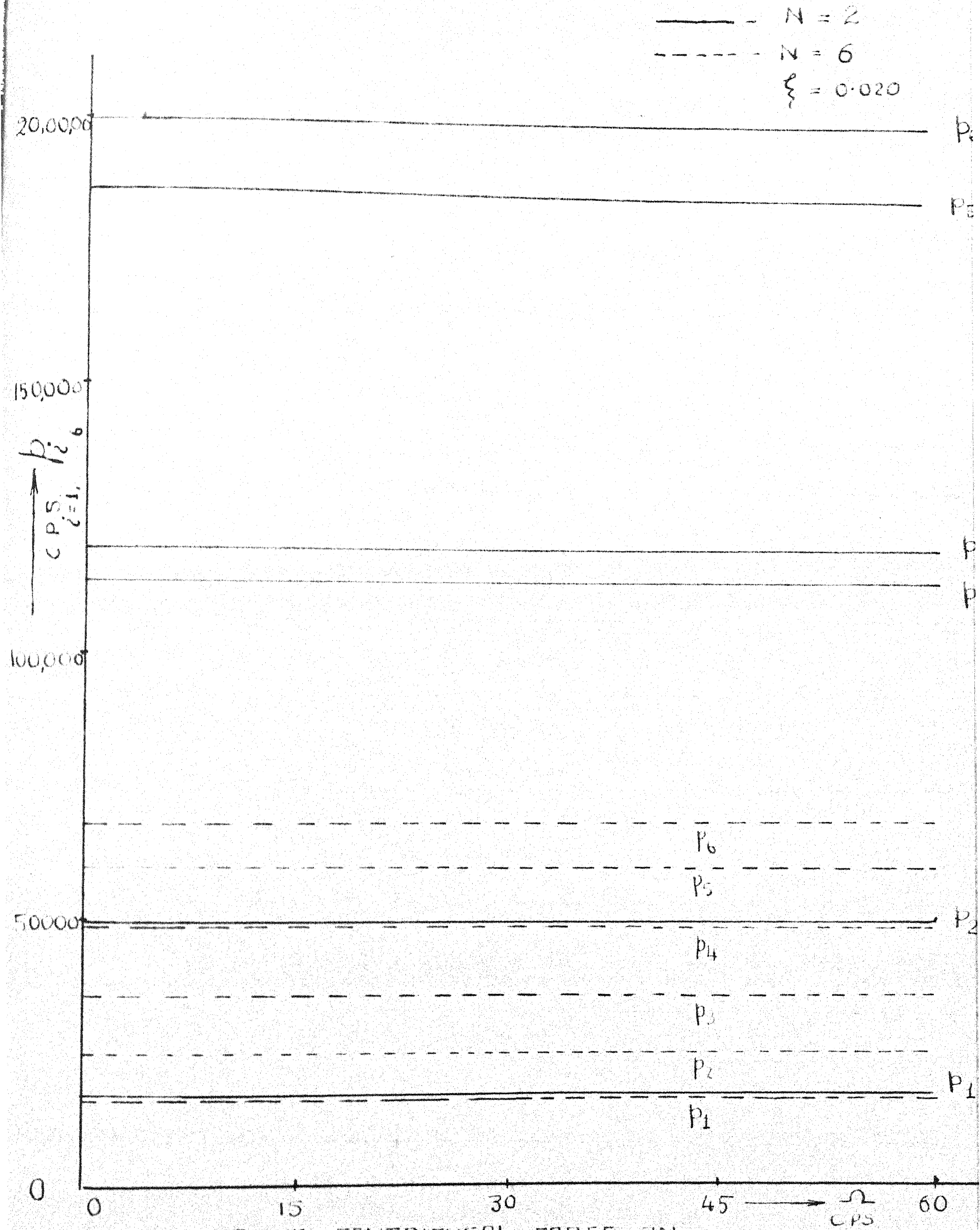
When the differences in natural frequencies p_i, $i > 2$ are plotted against N, we find that the effect of increasing number of blades is prominant in the beginning.

TABLE : 7

EFFECT OF NUMBER OF BLADES IN THE GROUP

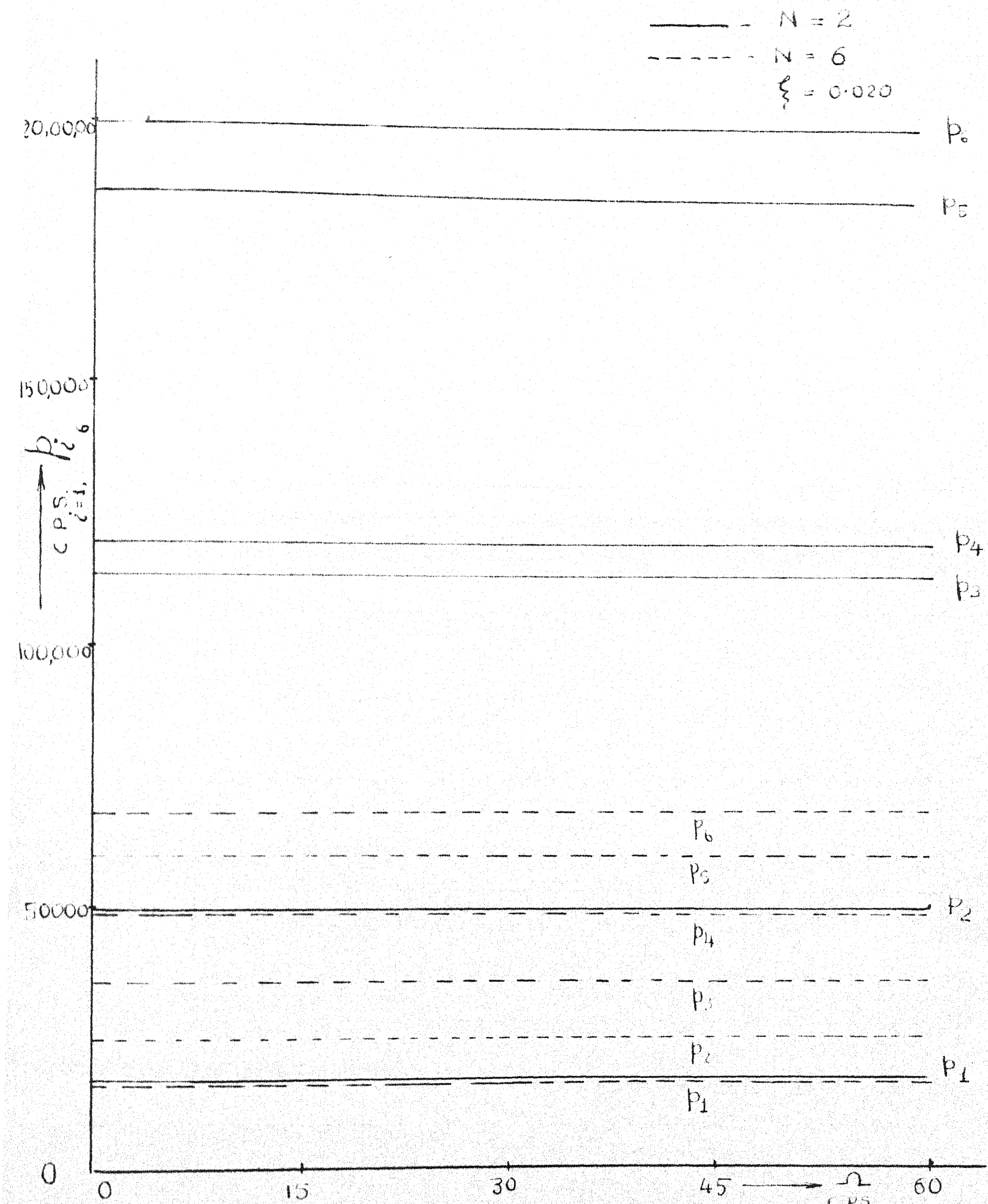
Here for an arbitrary ξ and any arbitrary order of frequency, the reduction in p is found for every additional balde that is grouped. This is carried out in both the types of mode. These reductions for arbitrary $\xi = 0.025$ and $p = p3$ are tabulated below:

N	Tangential Mode	Transverse - Torsional Mode
2	313504	60935
3	53001	5066
4	11280	4269
5	67795	2174
6	5368	1053



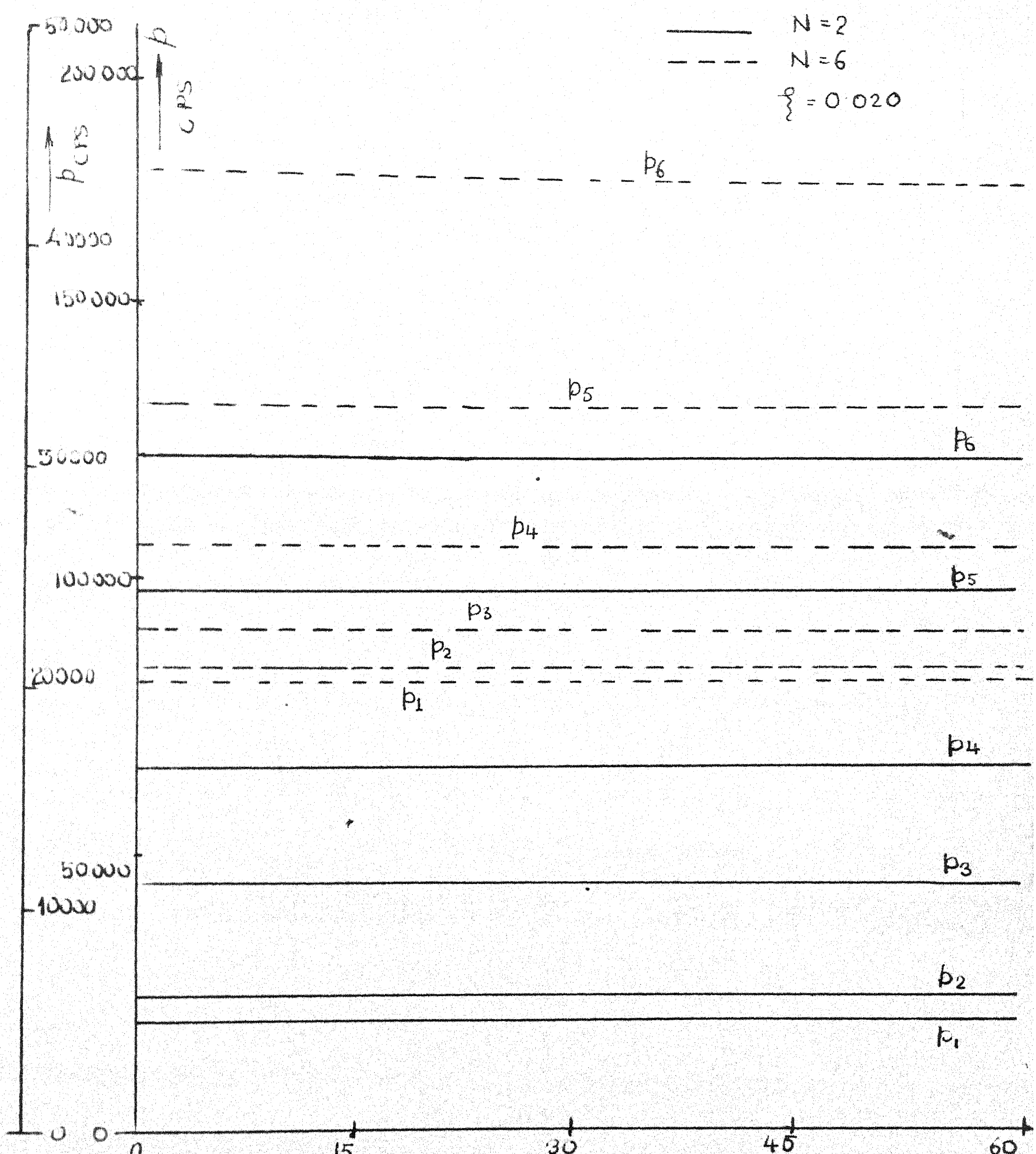
EFFECT OF CENTRIFUGAL FORCE ON TANGENTIAL VIBRATION MODE

Fig. 4.12



EFFECT OF CENTRIFUGAL FORCE ON TANGENTIAL VIBRATION MODE

FIG 4·1·a



EFFECT OF CENTRIFUGAL FORCE ON COUPLED TRASVERSE - TORSIONAL MODE OF VIBRATION

FIG 4.16

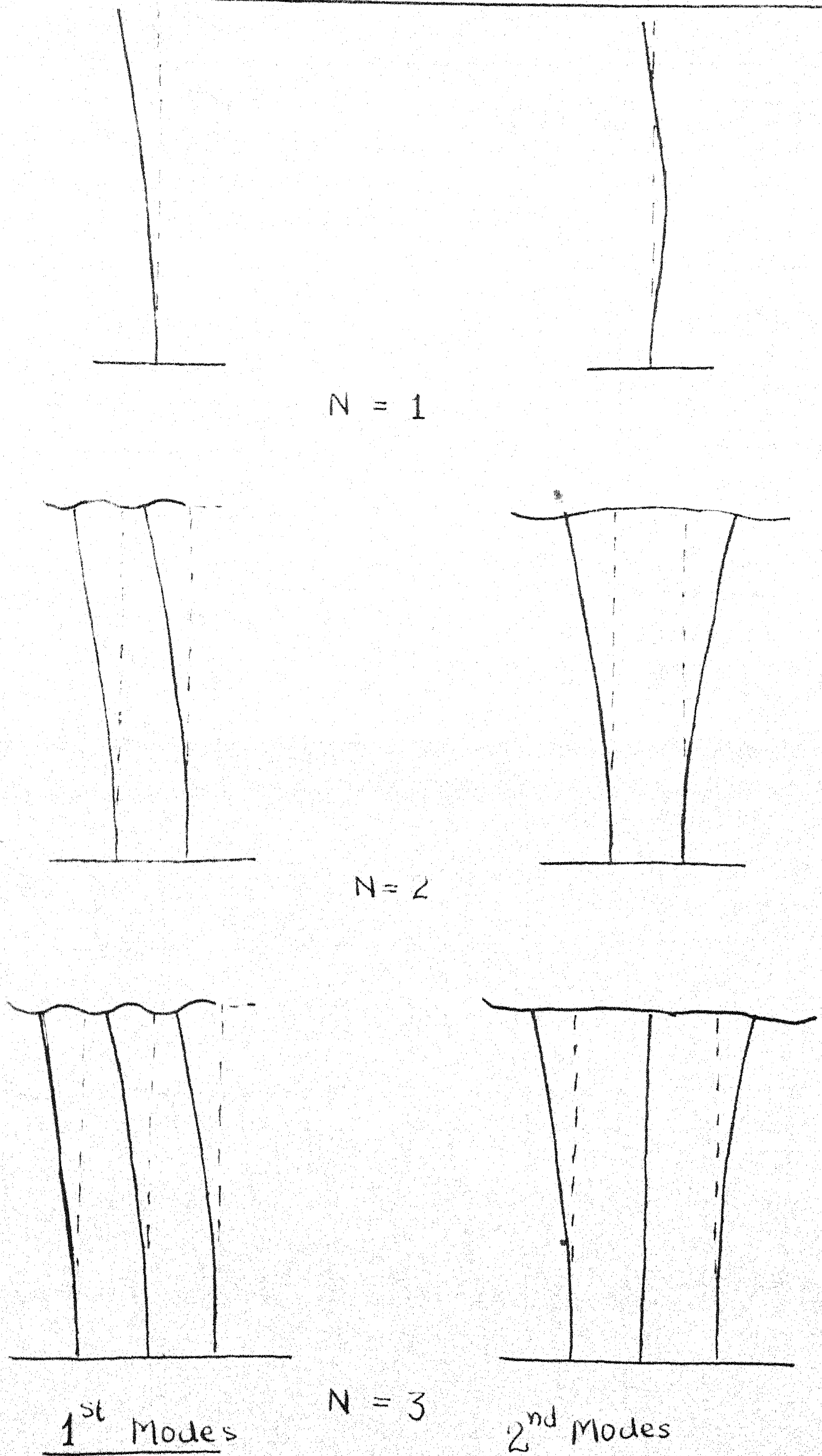
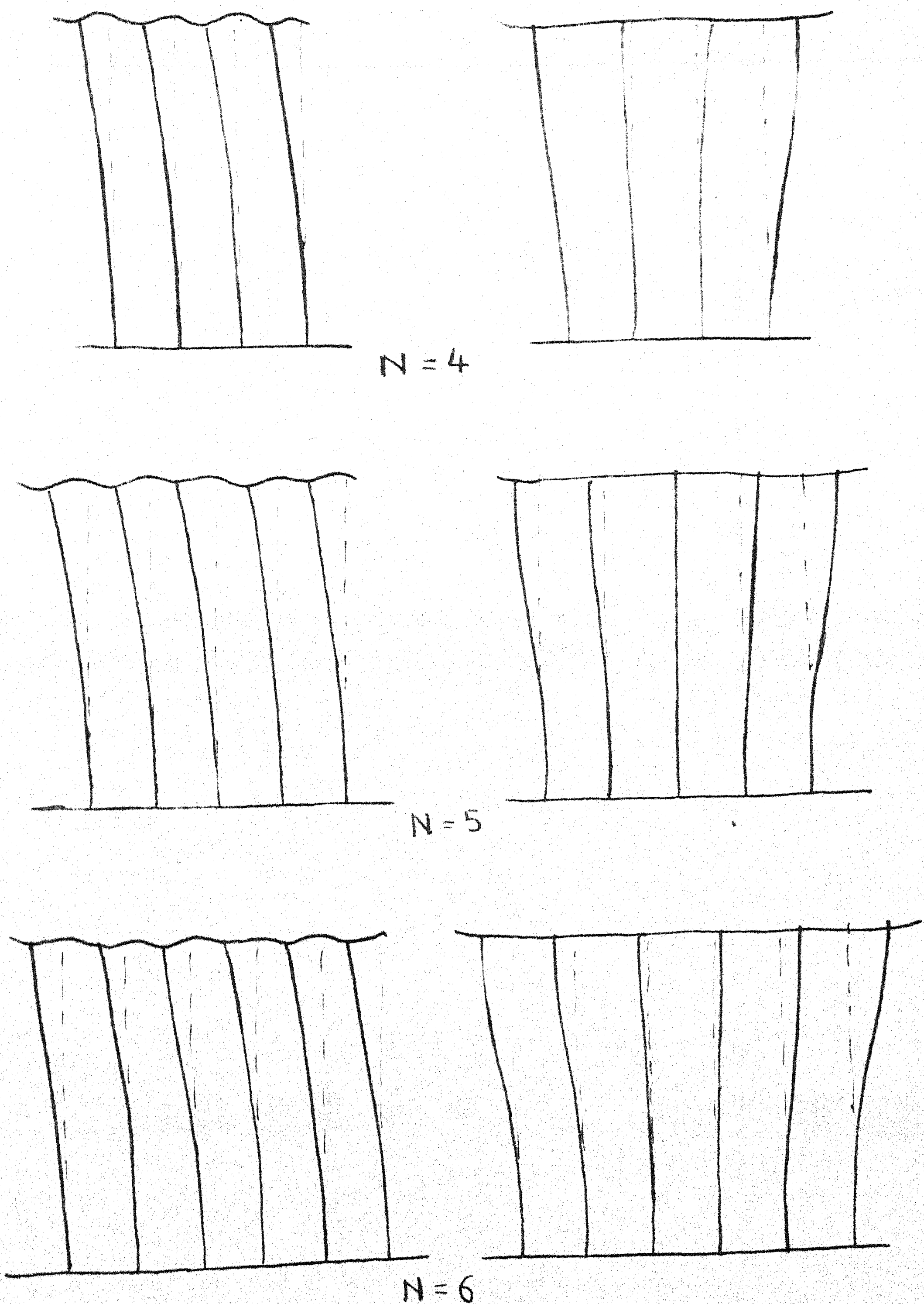


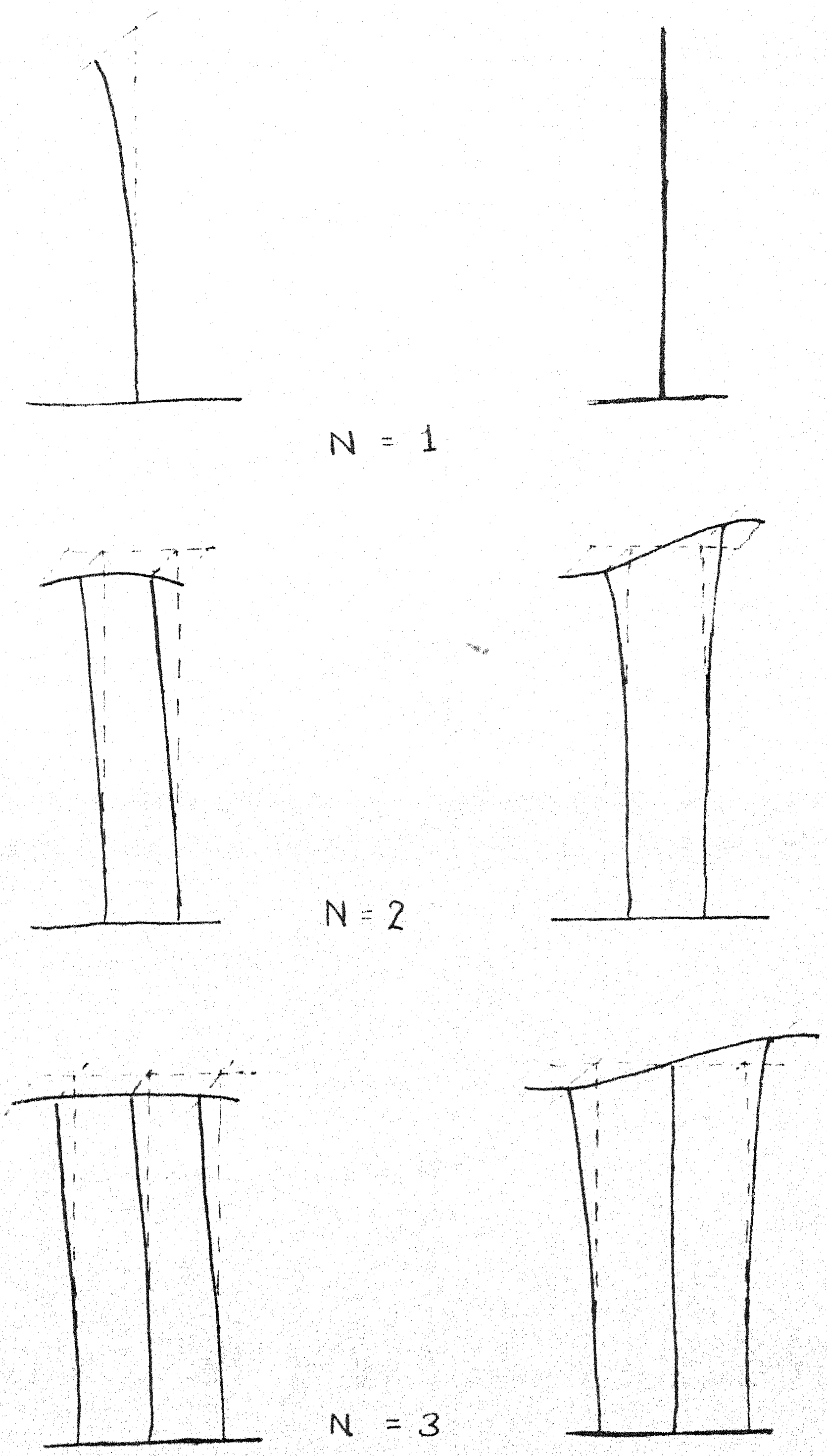
FIG 42.1

(Contd.)



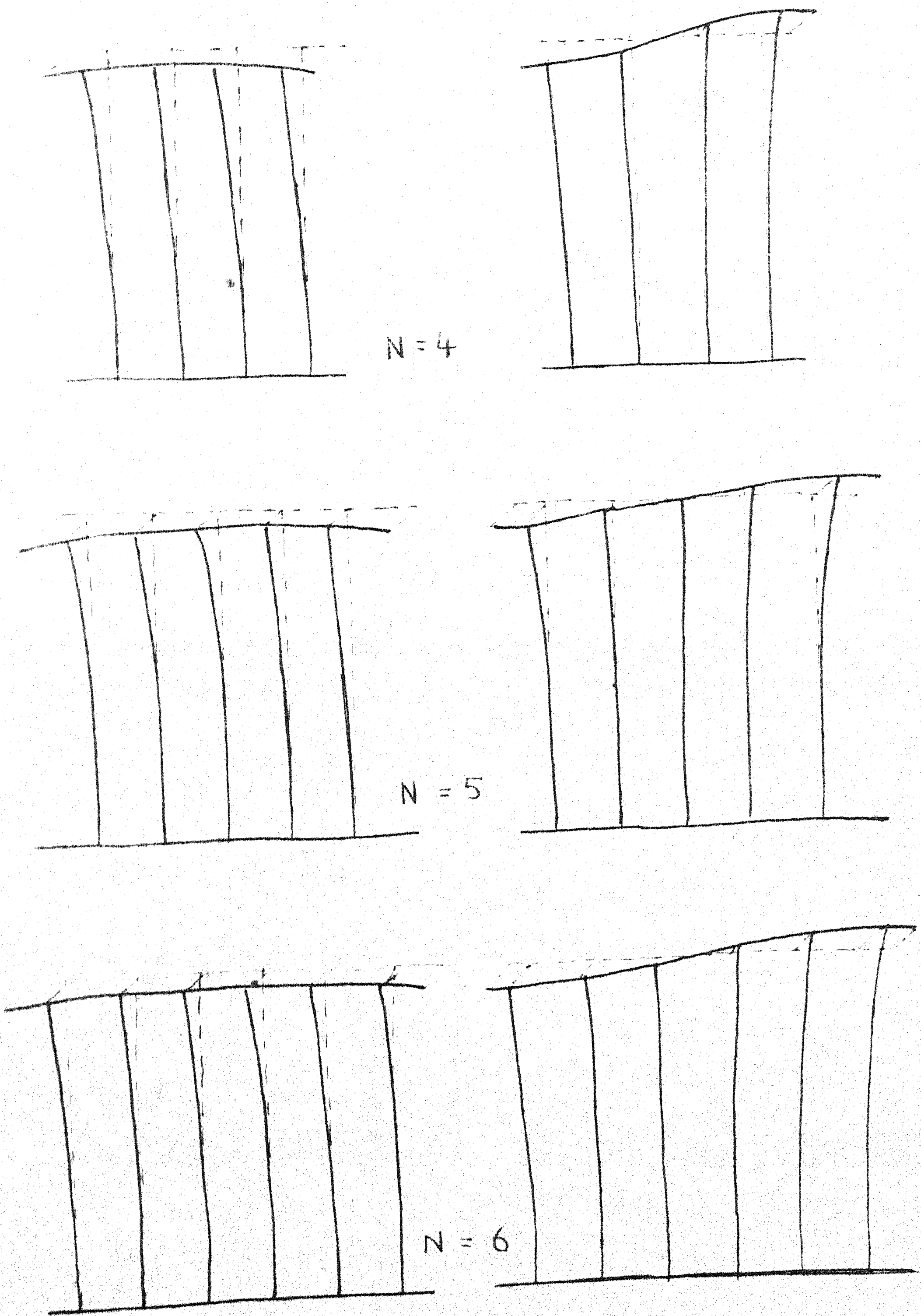
FIRST TWO MODE SHAPES FOR
TANGENTIAL VIBRATION

FIG. 4.21



1^{S} TWO MODE SHAPES (Contd.)

FIG 4.22



FIRST TWO MODE SHAPES OF COUPLED TRA.-TORS VIBRATIONS

FIG 4·2·2

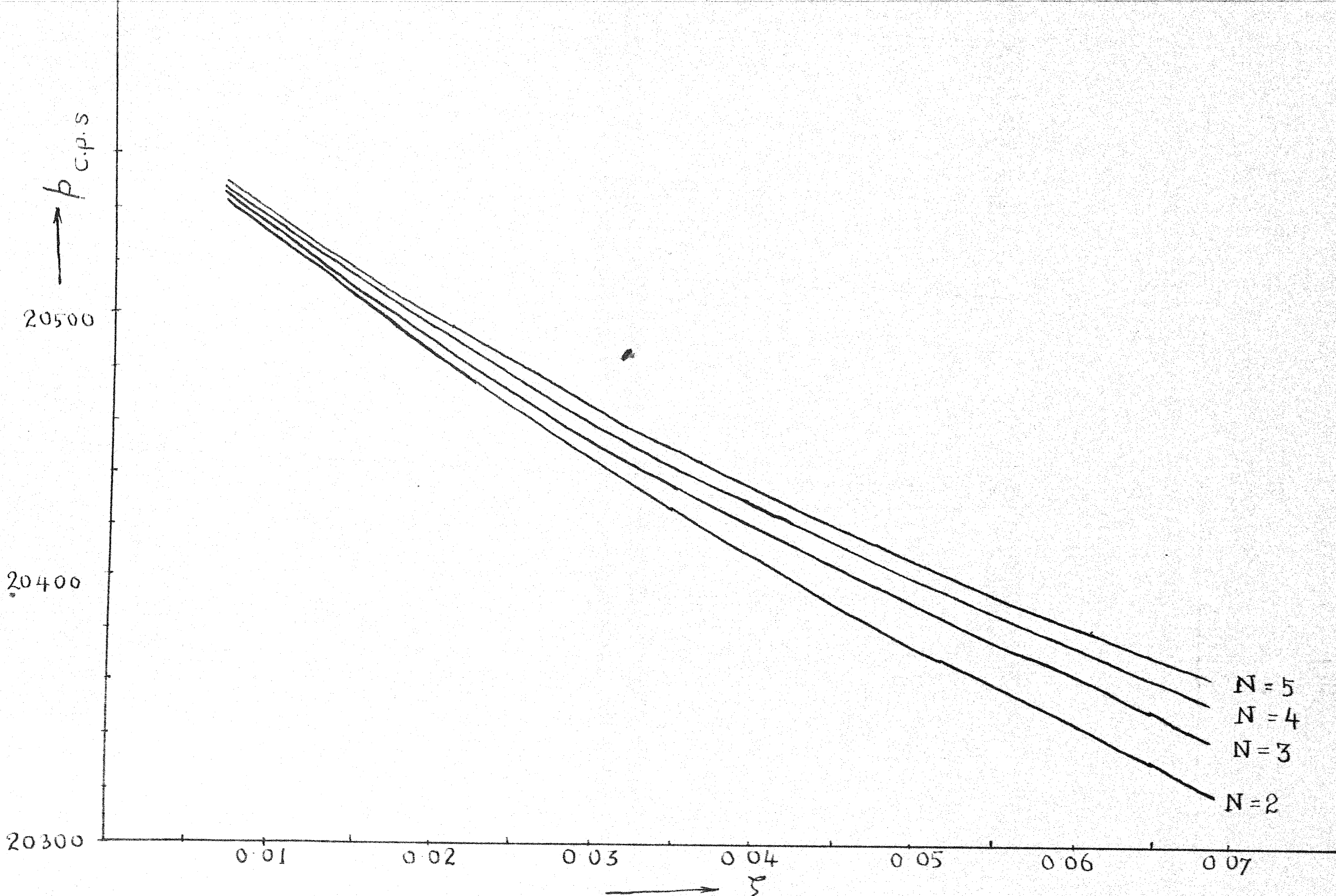
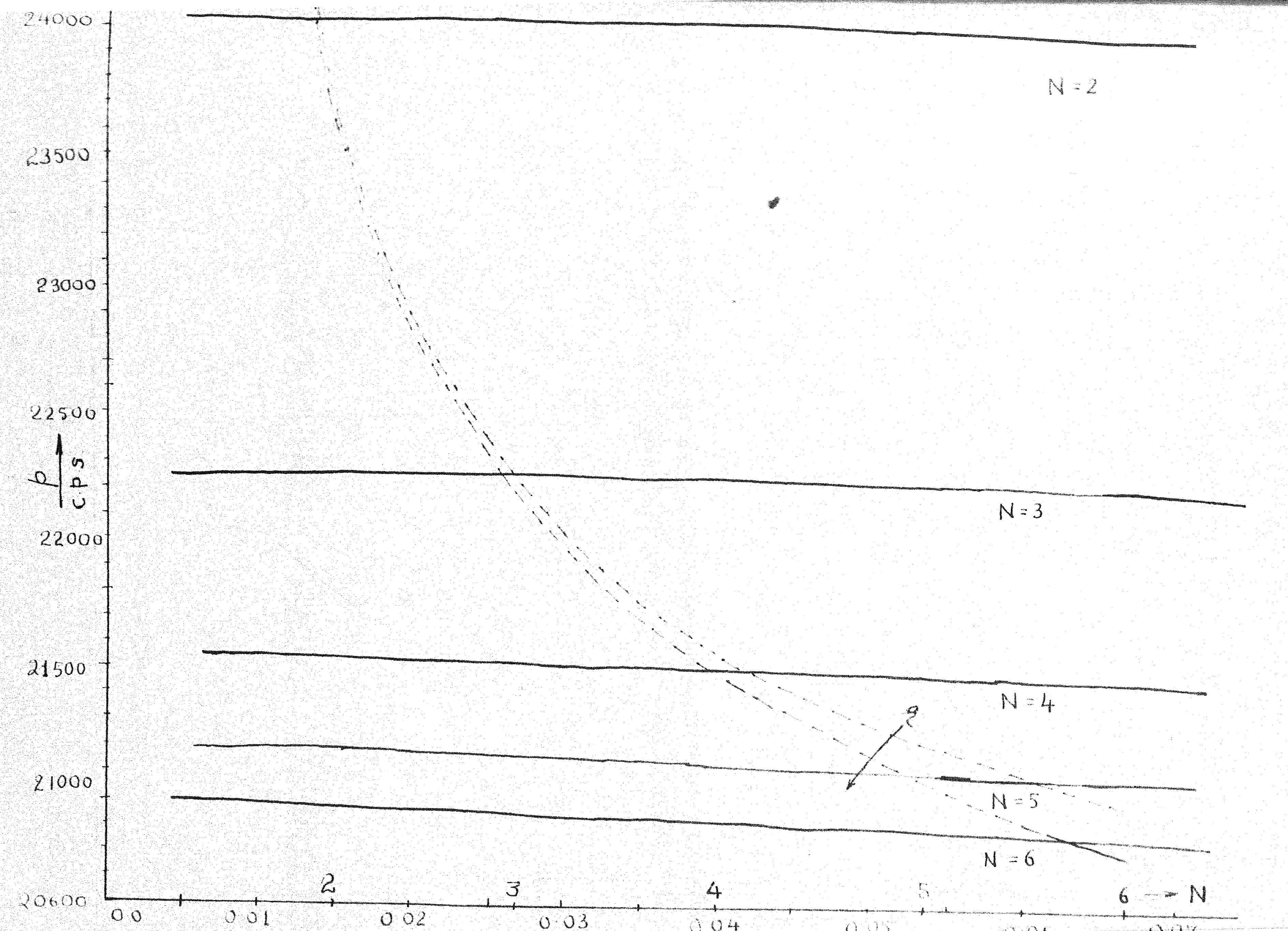
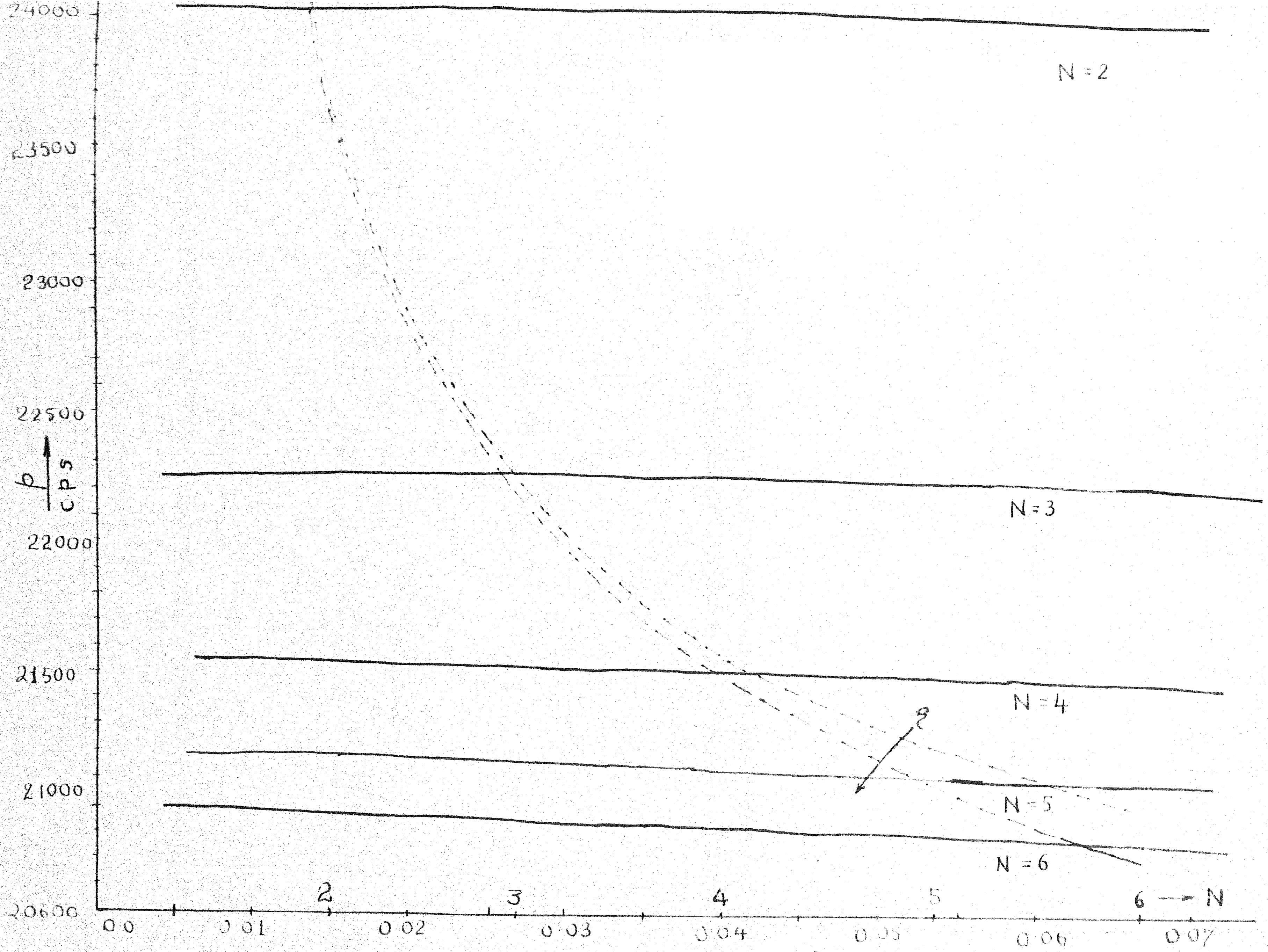


FIG 4·3·1

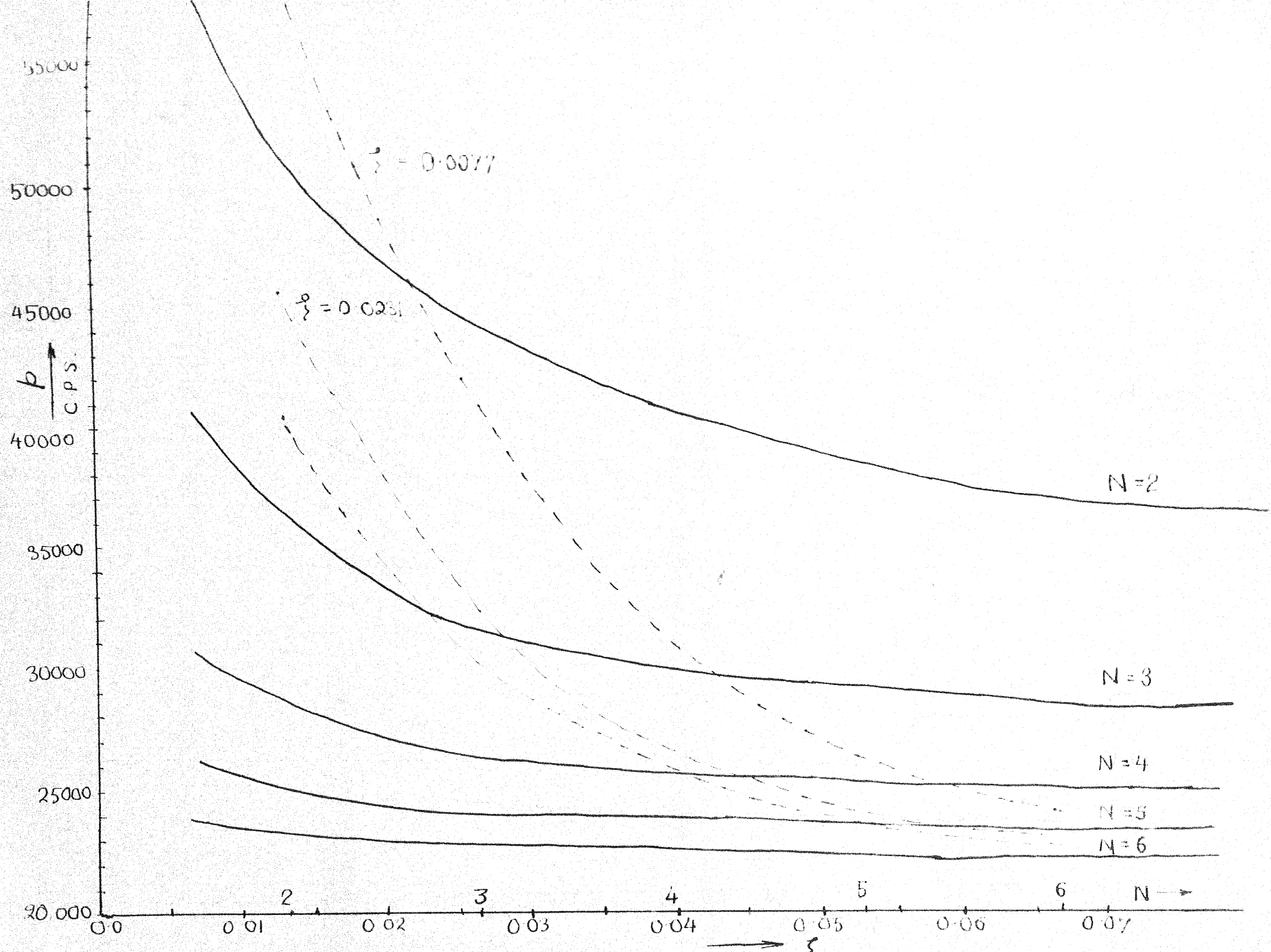


24000
23500
23000
22500
22000
21500
21000
20600
p
c.p.s
N = 2
N = 3
N = 4
N = 5
N = 6
g
2
3
4
5
6 → N
0.0
0.01
0.02
0.03
0.04



24000
23500
23000
22500
22000
21500
21000
20600
p
cps
N = 2
N = 3
N = 4
N = 5
N = 6
ξ
2
3
4
5
6 → N
0.0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
→ ξ

Fig 4.32



55000
50000
45000
40000
35000
30000
25000
20 000
p c.p.s.
$\xi = 0.0077$
$\xi = 0.0231$
N = 2
N = 3
N = 4
N = 5
N = 6
2
3
4
5
6
N
0.0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
ξ

FIG 4.3.3

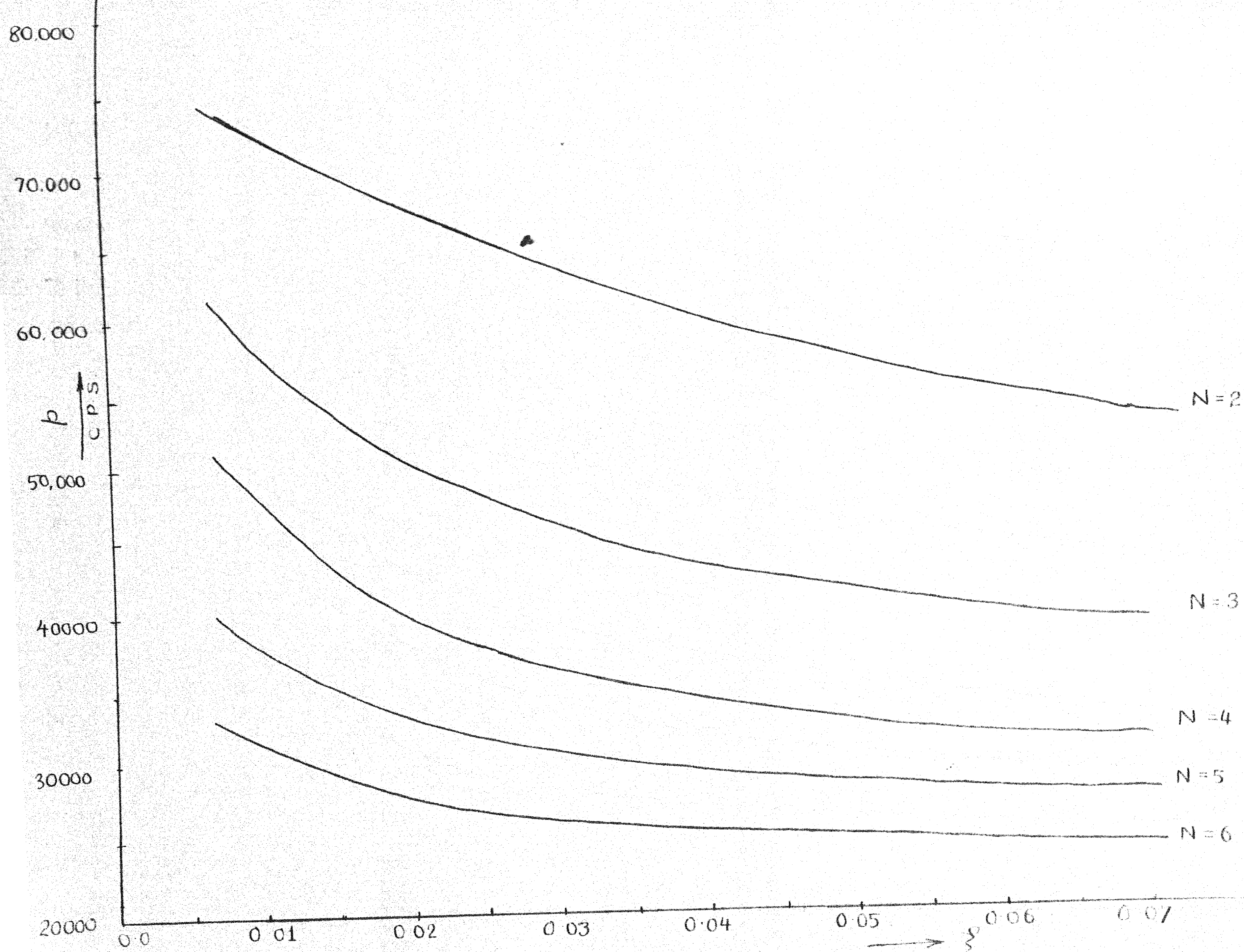

80.000
70.000
60,000
p
c p s
50,000
40000
30000
20000
0·0
0 01
0 02
0 03
0·04
0·05
0·06
0·07
ξ
N = 2
N = 3
N = 4
N = 5
N = 6

FIG. 4.3 4

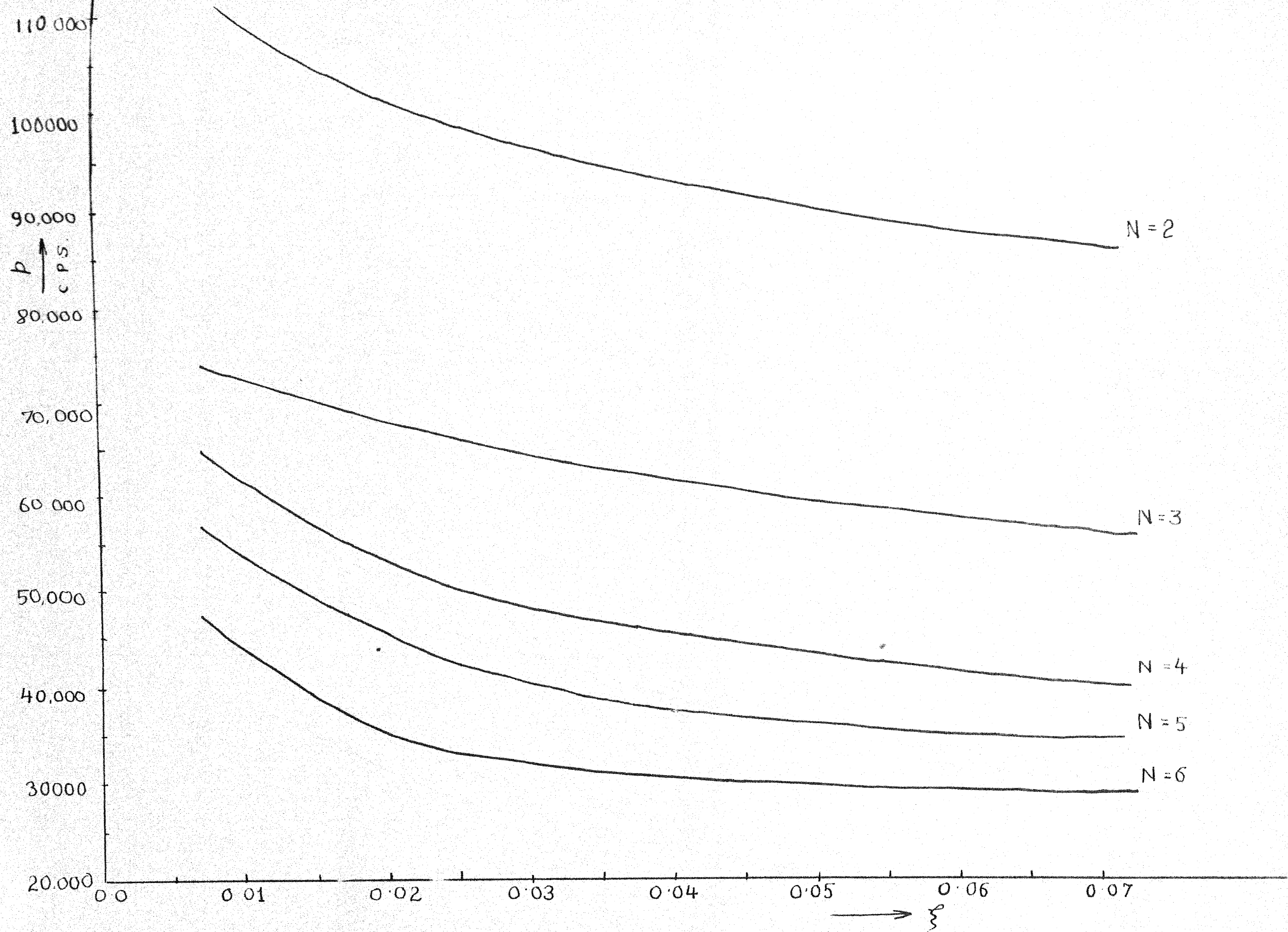
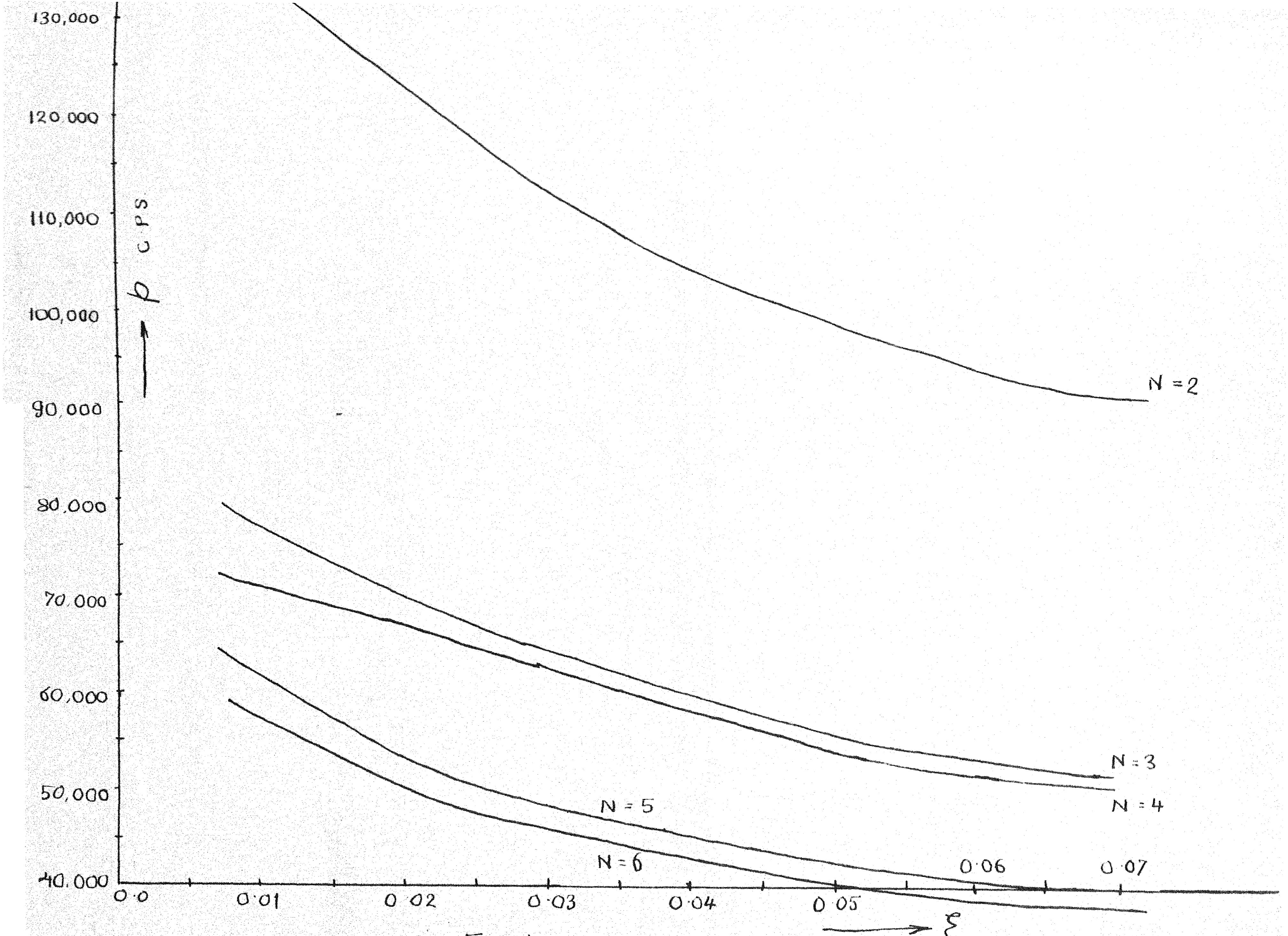
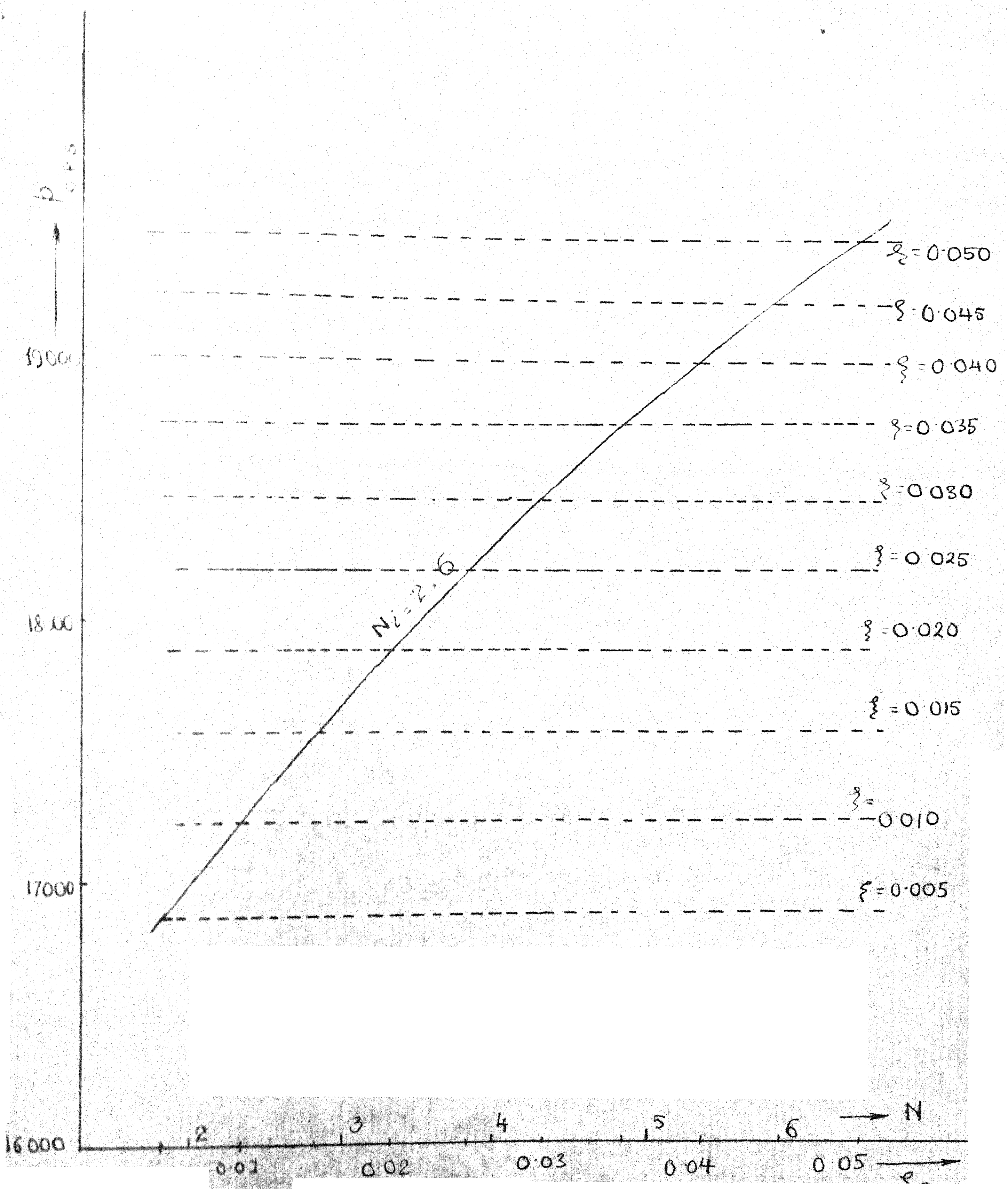

110 000
100000
90,000
80,000
70,000
60 000
50,000
40,000
30000
20.000
p
c p s.
0.0
0·01
0·02
0·03
0·04
0·05
0·06
0·07
ξ
N = 2
N = 3
N = 4
N = 5
N = 6

FIG 4·3·5



130,000
120,000
110,000
100,000
90,000
80,000
70,000
60,000
50,000
40,000
p c.p.s.
N = 2
N = 3
N = 4
N = 5
N = 6
0.0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
ξ

FIG. 4.3.6



p cps
19000
18000
17000
16000
ξ = 0·050
ξ = 0·045
ξ = 0·040
ξ = 0·035
ξ = 0·030
ξ = 0·025
ξ = 0·020
ξ = 0·015
ξ = 0·010
ξ = 0·005
Ni = 2.6
2
3
4
5
6
N
0·01
0·02
0·03
0·04
0·05
e

FIG 4·4·1

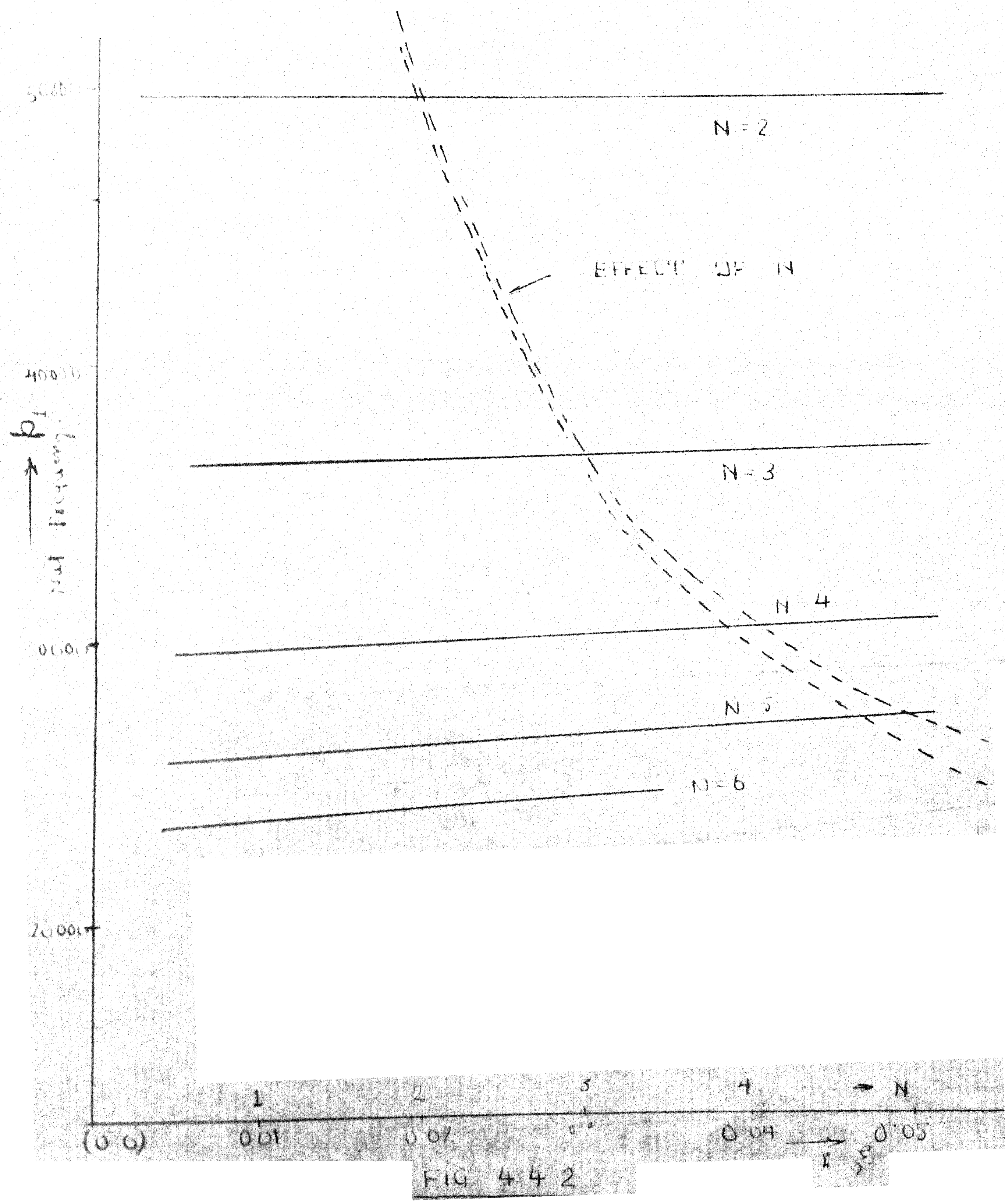
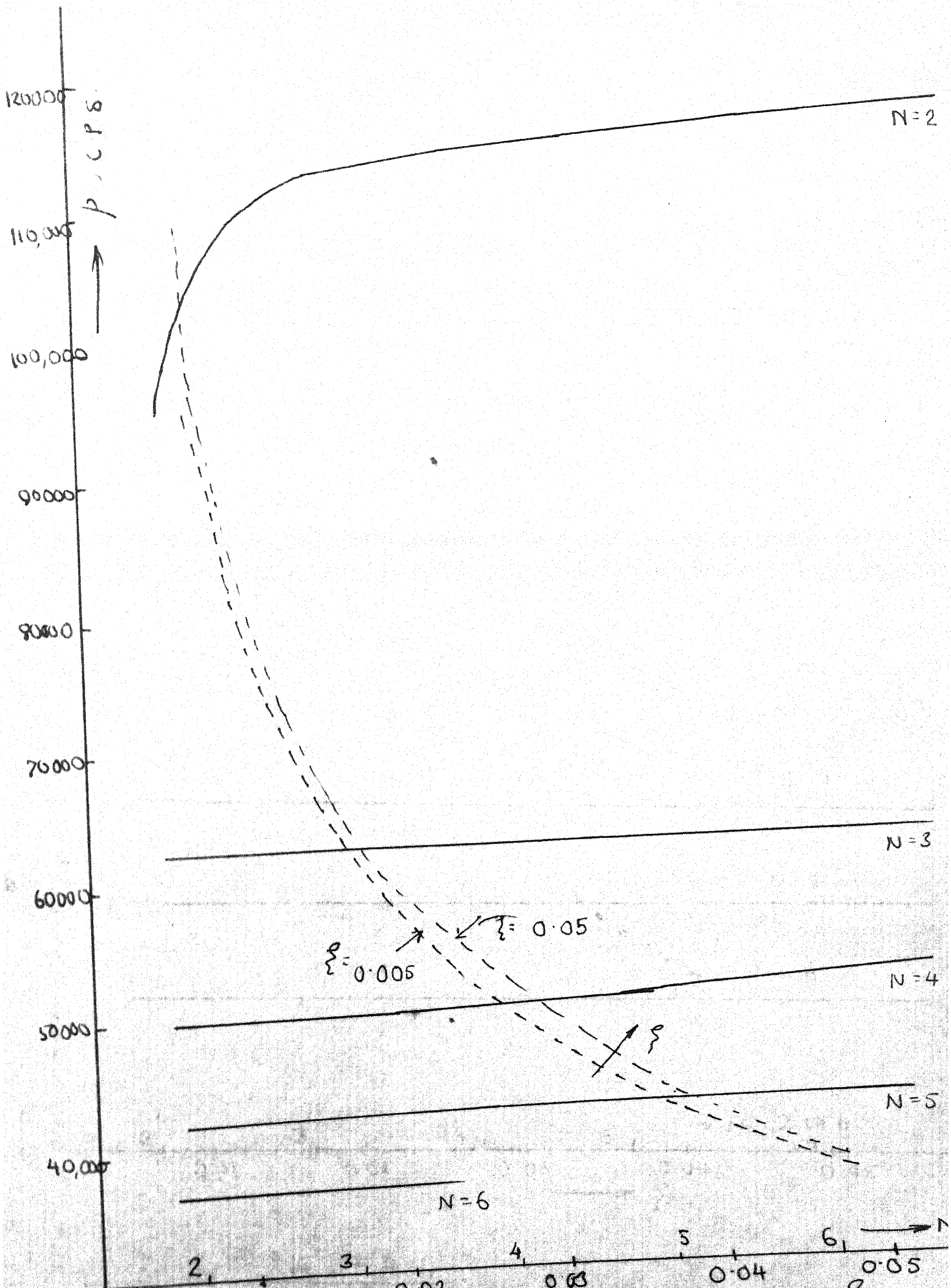

50000
40000
30000
20000
p_1
Net frequency
N = 2
EFFECT OF N
N = 3
N 4
N 5
N = 6
(0 0)
1
2
3
4
N
0·01
0·02
0·04
0·05

FIG 4 4 2



120000
110,000
100,000
90000
80000
70000
60000
50000
40,000
p, cps
N = 2
N = 3
N = 4
N = 5
N = 6
ξ = 0·005
ξ = 0·05
ξ
2
3
4
5
6
N
0·01
0·02
0·03
0·04
0·05
ξ

FIG 4·4·3

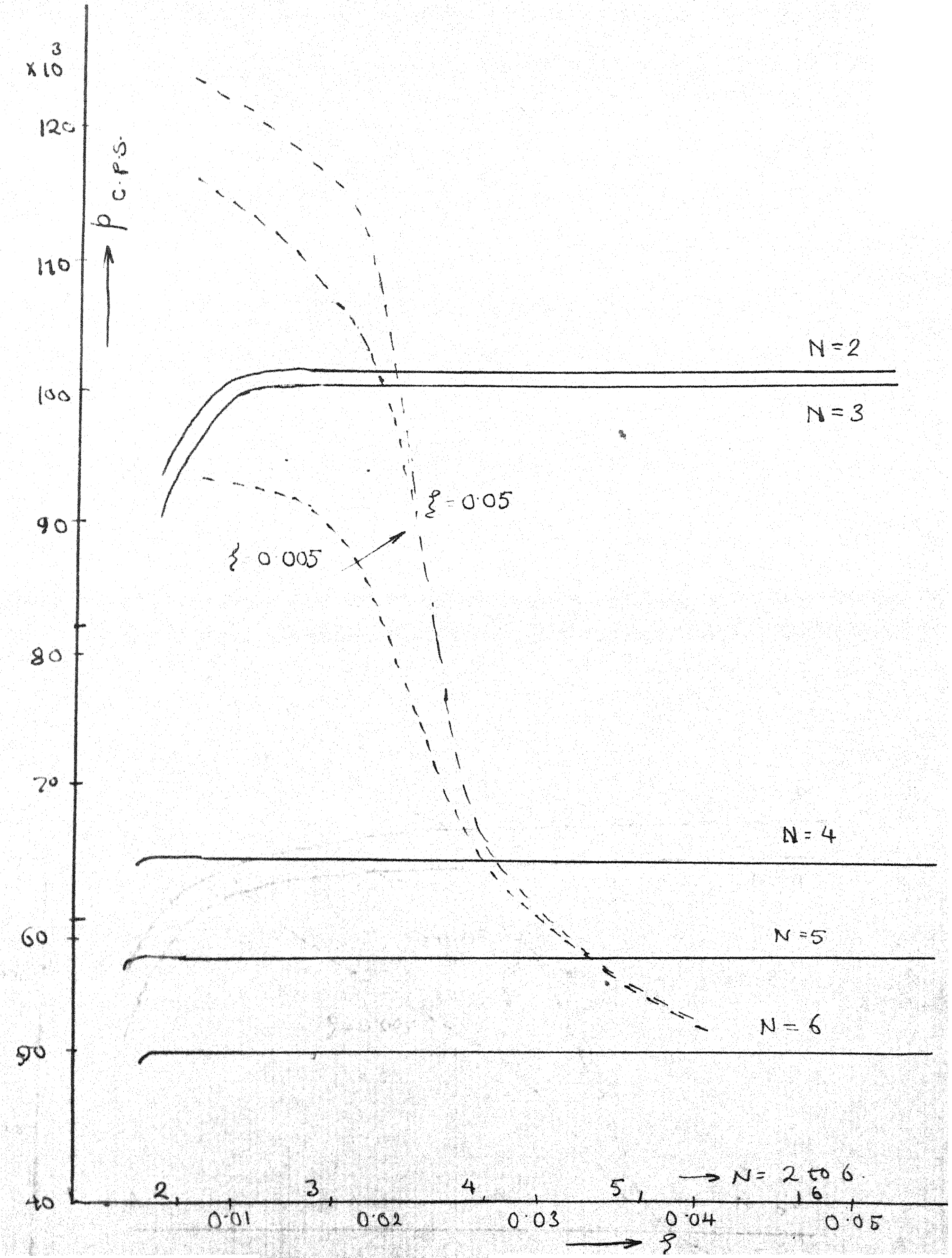
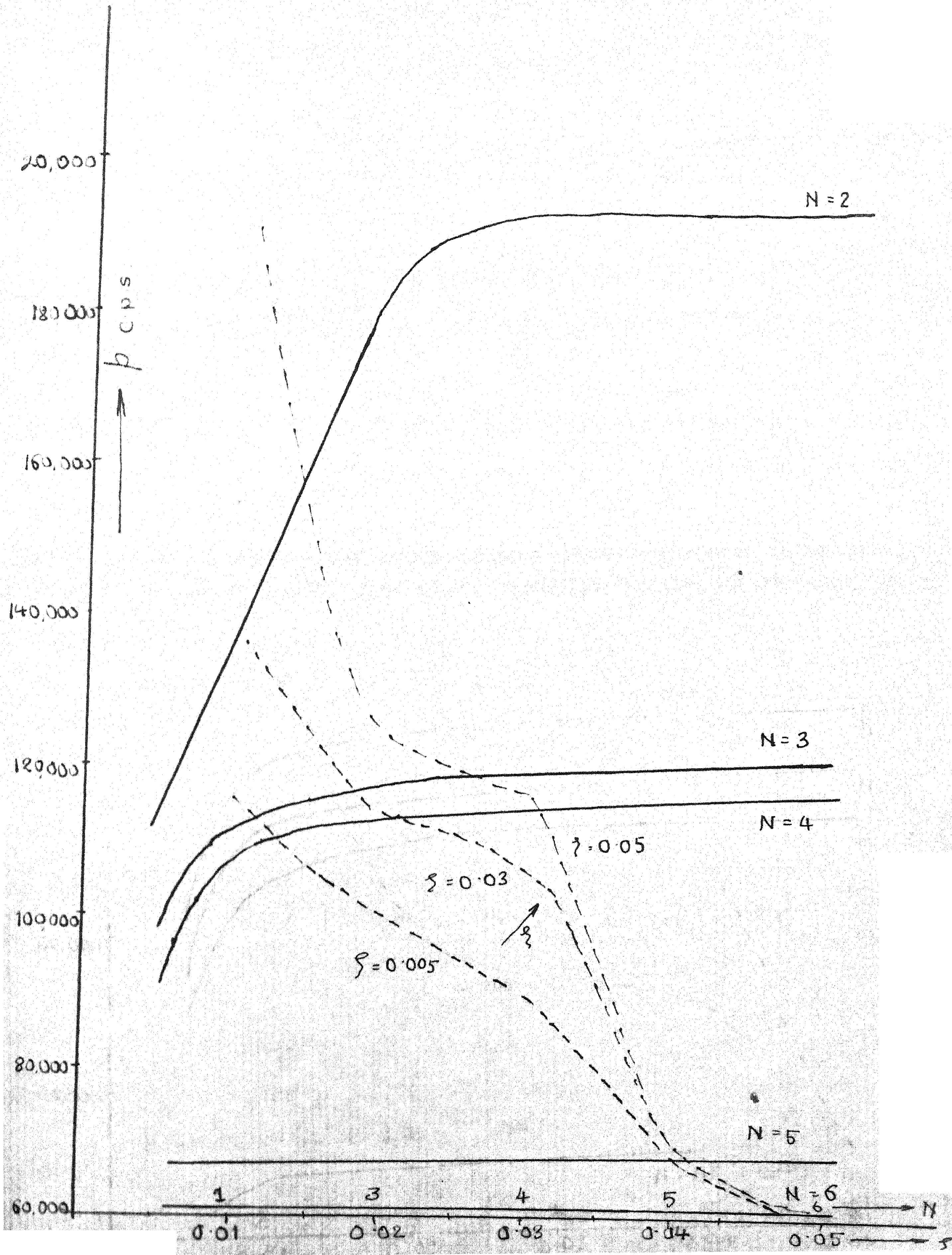

x 10^3
120
110
100
90
80
70
60
50
40
p c.p.s.
N = 2
N = 3
N = 4
N = 5
N = 6
ξ = 0·05
ξ 0·005
2
3
4
5
6
N = 2 to 6
0·01
0·02
0·03
0·04
0·05
ξ

FIG 4.4.4



20,000
180,000
160,000
140,000
120,000
100,000
80,000
60,000
p c p s
N = 2
N = 3
N = 4
N = 5
N = 6
ξ = 0·05
ξ = 0·03
ξ = 0·005
ξ
1
3
4
5
6
N
0·01
0·02
0·03
0·04
0·05
ξ

FIG 4.4.5

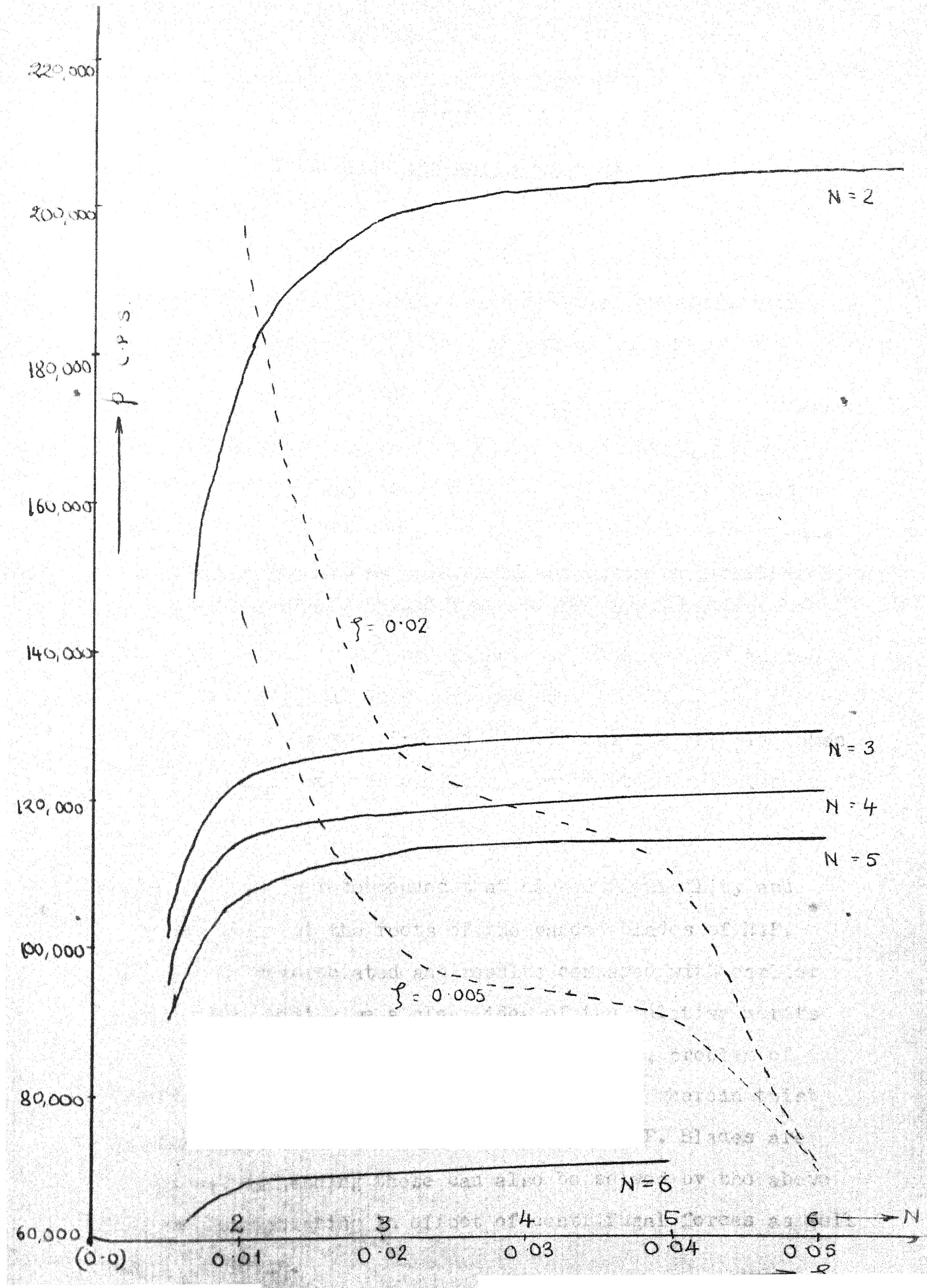

220,000
200,000
180,000
160,000
140,000
120,000
100,000
80,000
60,000
p c.p.s
N = 2
N = 3
N = 4
N = 5
N = 6
ρ = 0·02
ρ = 0·005
(0·0)
2
3
4
5
6
N
0·01
0·02
0·03
0·04
0·05
ρ

FIG 4·4·6

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

In the present work, the problem of banded blades is analysed for its vibration, tangential and transverse - torsional modes. Finite element method is used.

The results for a short blade show that centrifugal forces have neglegible effect whereas banding have appreciable effect. In addition to method of computing natural frequencies and mode shapes of banded blades, data has been calculated and presented which can be useful for the design of the turbine blade of H.P. Section. The relative merits of the method can not be presented due to the fact that only a limited work has been published for the vibrations of banded blades, in particular the example taken in the earlier work is different from that of the present work.

It is recommended that hinged flexibility and stagger angle at the roots of the banded blades of H.P. Section be incorporated and results compared with earlier work. This will give a clear idea of the relative merits of the two approaches. In addition to this, problem of banded blades of I.P. Section can be solved wherein twist of the blade can be incorporated. Since L.P. Blades are connected by lashing these can also be solved by the above method incorporating in effect of centrifugal forces as well as taper twist.

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APPENDIX A

EFFECT OF ROTOR SPEED ON THE VIBRATION FREQUENCIES FOR BANDED BLADES

In this appendix, the results for fundamental and higher order natural frequencies are tabulated for various rotor speeds. Two cases of banded groups having 2 and 6 blades are considered for $\xi = 0.02$ (corresponding $\xi = 0.231$) for tangential and coupled Transverse - Torsional modes of vibrations.

TABLE - A.1

TANGENTIAL MODE OF VIBRATION

($\xi = 0.020$)

	p \ c.p.s	0	15	30	45	60
	p_1	17893.2	17893.2	17893.2	17893.2	17893.2
	p_2	49930.2	49930.2	49930.2	49930.2	49930.2
	p_3	114221.4	114221.4	114221.4	114221.4	114221.4
N = 2	p_4	120678.1	120678.1	120678.1	120678.1	120678.1
	p_5	187136.5	187136.5	187136.5	187136.5	187136.5
	p_6	198738.5	198738.5	198738.5	198738.5	198738.5
	p_1	17893.2	17893.2	17893.2	17893.2	17893.2
	p_2	24225	24225	24225	24225	24225
	p_3	36777.7	36777.7	36777.7	36777.7	36777.7
N = 6	p_4	49921.8	49921.8	49921.8	49921.8	49921.8
	p_5	61201.4	61201.4	61201.4	61201.4	61201.4
	p_6	68807.4	68807.4	68807.4	68807.4	68807.4

TABLE - A.2

COUPLED TRANSVERSE - TORSIONAL MODE

(ξ = 0.0231)

	p \ c.p.s Ω	0	15	30	45	60
N = 2	p_1	20472.6	20472.6	20472.6	20472.6	20472.6
	p_2	24058.7	24058.7	24058.7	24058.7	24058.7
	p_3	45136.7	45136.7	45136.7	45136.7	45136.7
	p_4	66982.6	66982.6	66982.6	66982.6	66982.6
	p_5	99365.4	99365.4	99365.4	99365.4	99365.4
	p_6	133408.4	133408.4	133408.4	133408.4	133408.4
N = 6	p_1	20540.3	20540.3	20540.3	20540.3	20540.3
	p_2	20963.8	20963.8	20963.8	20963.8	20963.8
	p_3	22895	22895	22895	22895	22895
	p_4	26638.3	26638.3	26638.3	26638.3	26638.3
	p_5	33185.7	33185.7	33185.7	33185.7	33185.7
	p_6	48015.5	48015.5	48015.5	48015.5	48015.5

APPENDIX B

DATA FOR NATURAL FREQUENCIES OF VIBRATION OF BANDED BLADES

In this appendix values of natural frequencies for different modes for a stationary banded group of blades in both tangential and coupled transverse - torsional mode is given. The number of blades N is varied from 2 to 6. For every group, the band to blade stiffness ratio ξ is varied from 0.005 to 0.06. The values of natural frequencies p's are tabulated below.

The data is presented in three sections. Sections B.1 and B.2 give results for tangential and coupled transverse - torsional modes respectively and in Section B.3 coordinates for first two eigen vectors for both modes for N = 2 to 6 are given. In each sections B.1 and B.2 six tables are given for first six natural frequencies. The corresponding natural frequency p for the single blade is given on the top of the table.

TABLE B1.1

FUNDAMENTAL NATURAL FREQUENCY

($p_{\text{single blade}}$ = 24,304.34 c.p.s)

ξ \ N	2	3	4	5	6
0.005	16879.25	16879.25	16879.25	16879.25	16879.25
0.010	17238.36	17238.36	17238.36	17238.36	17238.36
0.015	17575.61	17575.61	17575.61	17575.61	17575.61
0.020	17893.25	17893.25	17893.25	17893.25	17893.25
0.025	18193.10	18193.10	18193.10	18193.10	18193.10
0.030	18476.72	81476.72	18476.72	18476.72	18476.72
0.035	18745.49	18745.49	18745.49	18745.49	18745.49
0.040	19000.60	19000.60	19000.60	19000.60	19000.60
0.045	19243.13	19243.13	19243.13	19243.13	19243.13
0.050	19474.03	19474.03	19474.03	19474.03	19474.03

TABLE B.1.2

SECOND HIGHER NATURAL FREQUENCY

(p, single blade = 152,39.30)

ζ \ N	2	3	4	5	6
0.005	49781.5	36404.0	29693.0	25882.0	23530
0.010	49836.2	36539.7	29875.0	26100.8	23770
0.015	49880.3	36662.5	30046.7	26307.7	24003
0.020	49930.2	36779.0	30209.9	26504.3	24225
0.025	49977.2	36889.8	30365.3	26691.3	24435
0.030	50021.6	36995.4	30513.4	26869.5	
0.035	50063.8	37096.3	30654.9	27039.6	
0.040	50104.0	37192.6	30790.2	27202.1	
0.045	50142.3	37284.8	30919.6	27357.5	
0.050	50178.8	37373.0	31043.6	27506.2	

TABLE B.1.3

THIRD HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 427,725.9)

ζ \ N	2	3	4	5	6
0.005	90568.1	61198,4	49772.7	41848.6	36385.4
0.010	110196.7	61201.8	49841.3	41968.74	36557.9
0.015	113069.0	61211.4	49892.5	42029.5	36661.6
0.020	114221.4	61220.3	49940.6	42145.36	36777.7
0.025	115021.4	61228.9	49986.1	42226.8	36878.0
0.030	115687.2	61236.9	50029.2	42304.3	
0.035	116283.4	61244.6	50070.2	42378	
0.040	116836.3	61251.8	50123.7	42433.8	
0.045	117358.2	61258.3	50161.05	42500.8	
0.050	117856.2	61264.8	50196.6	42564;9	

TABLE B.1.4

FOURTH HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 845584.6)

ξ \ N	2	3	4	5	6
0.005	93529.6	90412.6	65532.0	57009.3	49794.7
0.010	116757.6	109724.8	65539.4	57008.2	49817.21
0.015	119676.0	112522.6	65521.1	57024.4	49886.1
0.020	120678.1	113656.1	65542.4	57057.5	49929.8
0.025	121664.0	114443.4	65154.3	57077.0	49970.4
0.030	122173.1	115100.3	65544.6	57082.2	
0.035	122762.4	115688.1	65545.1	57098.9	
0.040	123322.3	116232.7	65527.0	57135.7	
0.045	123854.3	116748.0	65527.0	57151.3	
0.050	124358.7	117238.6	65528.3	57166.9	

TABLE B.1.5

FIFTH HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 1,403,307)

ξ \ N	2	3	4	5	6
0.005	115780.6	99523.1	90439.1	67581.34	61170.8
0.010	137410.1	113816.0	109766.0	67610.94	61185.5
0.015	101132.1	116823.0	112604.0	67619.9	61195.5
0.020	187136.5	118007.0	113856.7	67683.3	61201.4
0.025	192635.4	118838.0	114600.9	67602.4	61204.3
0.030	195661.8	119561.6	115194.0	67616.4	
0.035	195246.2	120190.6	115808.6	67619.4	
0.040	195757.9	120769.6	116823.5	67618.5	
0.045	196253.3	121314.0	116801.8	67618.8	
0.050	196556.0	121821.5	117481.7	67618.0	

TABLE B.1.6

SIXTH HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 2,333,891)

N	2	3	4	5	6
0.005	146532.0	101088.4	94487.9	90088.8	68113.5
0.010	183257.3	124805.5	113797.4	108770.1	68185.0
0.015	193371.7	105618.2	116219.6	111281.9	68782.7
0.020	198738.5	126493.2	117729.4	111957.7	68003.4
0.025	211326.5	127153.0	118468.5	113389.4	68807.4
0.030	213876.4	127658.0	119392.0	113648.2	69487.3
0.035	214169.2	128182.6	124392.1	114127.3	
0.040	215561.6	128702.1	120039.7	114437.5	
0.045	210359.9	129176.8	121120.5	114537.0	
0.050	216638.4	129654.5			

TABLE B.2.1

FUNDAMENTAL NATURAL FREQUENCY

($p_{\text{single blade}}$ = 30184.29)

ξ \ N_1	2	3	4	5	6
0.0077	20541.5	20543.5	20547.2	20549.6	20550.5
0.0231	20472.6	20478.5	20484.8	20489.4	20540.3
0.0385	20413.4	20423.9	20432.8	20438.9	20281.2
0.0539	20363.2	20378.3	20389.5	20397.0	20408.8
0.0691	20320.6	20340.2	20353.7	20362.4	20378.37

TABLE B.2.2

SECOND HIGHER NATURAL FREQUENCY

($p_{\text{single blade}}$ = 36793.8)

ξ \ N	2	3	4	5	6
0.0071	24021.4	22257.3	21545.3	21195.7	21000.84
0.0231	24058.7	22272.2	21538.0	21173.9	20963.8
0.0385	24068.8	22272.0	21523.5	21149.8	20921.7
0.0539	24047.4	22253.2	21499.3	21121.3	20885.0
0.0691	23996.4	22216.7	21465.7	21088.4	20867.5

TABLE B.2.3

THIRD HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 116071.2)

ξ \ N	2	3	4	5	6
0.0077	58068.3	40696.1	30719.8	26230.2	24052.0
0.0231	45136.7	32182.2	26687.9	24242.6	22895.0
0.0385	40553.5	30070.2	25801.8	23827.5	22674.85
0.0539	38184.7	29076.6	25391.6	23630.6	22458.3
0.0691	36713.1	28465.2	25127.1	23493.1	22281.5

TABLE B.2.4

FOURTH HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 189375.9)

ξ_1 \ N	2	3	4	5	6
0.0077	74951.2	61818.0	51194.3	40380.5	33344.61
0.0231	66982.6	48814.4	38378.25	31611.26	26638.3
0.0385	60954.8	43568.7	34674.14	29449.3	25453.79
0.0539	56574.4	40751.6	32869.40	28441.69	24723.0
0.0691	53326.3	38975.39	31767.11	27828.6	24025.7

TABLE B.2.5

FIFTH HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 210847.0)

ξ \ N	2	3	4	5	6
0.0077	113539.6	73071.6	64848.8	56802.3	47922.4
0.0231	99365.4	66917.5	50700.6	42554.8	33180.7
0.0385	94108.3	62109.4	45162.9	37885.09	30825.3
0.0539	90439.2	58549.19	42122.4	35537.9	29507.2
0.0691	86805.3	55746.0	40187.2	34091.7	27953.4

TABLE B.2.6

SIXTH HIGHER NATURAL FREQUENCY

($p_{single\ blade}$ = 304903.2)

ξ \ N	2	3	4	5	6
0.0071	141927.7	79056.8	71051.1	65181.3	59839.8
0.0231	133408.4	67621.5	66762.58	51772.0	48015.5
0.0385	105666.4	60037.49	59062.13	96099.5	43672.3
0.0539	97314.15	54960.8	53795.64	42941.15	40012.7
0.0691	90988.05	51243.5	50155.34	40917.6	37833.25

TABLE B.3

MODE SHAPES

(a) First two eigen vectors calculated for tangential modes of vibrations of banded blades N = 2 to 6 are given below for = 0.025

a) N = 2

$V_1 \equiv$ { 0.041, 0.447, 0.149, 0.758, 0.300, 0.937, 0.472, 1.000, 0.647,
0.041, 0.447, 0.149, 0.758, 0.300, 0.937, 0.472, 1.000, 0.647,
0.662, -0.493, 0.662, -0.493, 0.662, -0.493 }

$V_2 \equiv$ { 0.067, 0.698, 0.222, 1.000, 0.400, 0.973, 0.554, 0.766, 0.672,
-0.067, -0.698, -0.222, -1.000, -0.400, -0.973, -0.554, -0.766, -0.672, -
.803, -0.348, .000, -0.000, -0.803, 0.348 }

b) N = 3

$V_1 \equiv$ { 0.041, 0.448, 0.150, 0.758, 0.301, 0.938, 0.473, 1.000, 0.647,
0.041, 0.448, 0.150, 0.758, 0.301, 0.938, 0.473, 1.000, 0.647,
0.041, 0.448, 0.150, 0.758, 0.301, 0.938, 0.473, 1.000, 0.647,
0.662, -0.494, 0.662, -0.494, 0.662, -0.494, 0.067, 0.699, 0.222,

$V_2 \equiv$ { 0.054, 0.577, 0.188, 0.901, 0.358, 1.000, 0.531, 0.947, 0.689,
0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000,
-0.054, -0.577, -0.188, -0.901, -0.358, -1.000, -0.531, -0.947, -0.689, -
0.758, -0.494, 0.379, -0.231, -0.379, 0.231, -0.758, 0.462 }

$N = 4$

$V_1 \equiv \{A, A, A, A, B, B, B, B, B.\}$

Where $A = \{0.041, 0.447, 0.149, 0.758, 0.300, 0.937, 0.472, 1.000, 0.672, 0.969$

$B = \{0.662, -0.093\}$

$V_2 \equiv \{A, B, -B, -A, ^{?}$ $0.735, -0.492, 0.519, -0.348, 0.000, 0.000, -0.519, 0.348, -0.7351, + 0.492\}$

Where $A \equiv \{0.049, 0.532, 0.175, 0.859, 0.3419, 0.999, 0.519, 1.000, 0.681, 0.935\}$

$B = \{0.026, 0.220, 0.072, 0.355, 0.141, 0.414, 0.214, 0.414, 0.285, 0.3873\}$

$N = 5$

$V_1 \equiv \{A, A, A, A, A, B, B, B, B, B, B\}$

Where A = as detailed for V_1 in the case $N = 4$

B = as detailed for V_1 in the case $N = 4$

$V_2 = \{A, B, 0, -A, -B\}$

Where $A \equiv \{0.0466, 0.500, 0.165, 0.820, 0.325, 0.976, 0.501, 1.000, 0.673, 0.948\}$

$B \equiv \{0.028, 0.309, 0.1029, 0.507, 0.201, 0.603, 0.309, 0.618, 0.416, 0.5859\}$

$0 \equiv \{0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0\}$

$N = 6$

$$V_1 = \{A, A, A, A, A, A, B, B, B, B, B, B, B\}$$

Where $A \equiv$ as defined for V_1 in the case $N = 4$

$B \equiv$ as defined for V_2 in the case $N = 4$

$$V_2 \equiv \{A, B, C, -C, -B, -A, 0.069, -0.493, 0.600, -0.427, 0.346, -0.246, 0.0, 0.0, -0.346, 0.246, -0.600, 0.427, -0.693, 0.494\}$$

Where $A \equiv \{0.045, 0.483, 0.160, 0.89, 0.317, 0.964, 0.492, 1.00, 0.665, 0.954\}$

$B \equiv \{0.032, 0.353, 0.117, 0.586, 0.232, 0.705, 0.360\ 0.732, 0.486, 0.699\}$

$C \equiv \{0.012, 0.129, 0.0429, 0.214, 0.0851, 0.258, 0.131\ 0.267, 0.178, 0.255\}$

B = 0.00, 0.00, 0.000 ---------, 0.00

N = 4

V_1 = A, A, A, A, -0.618, 1.000, -0.09, -0.601, 1.000,
-0.001, -0.601, 0.999, -0.000, -0.601, 1.000, 0.001
-0.618, 1.000, 0.090

Where A = -0.053, 0.463, 0.000, -0.192, 0.768, 0.000
-0.381, 0.930, 0.000, -0.593, 0.976, 0.000

V_2 = A, B, -B, -A, -0.652, 1.000, -0.167, -0.438, 0.707
-0.703, 0.000, 0.000, -0.991, 0.438, -0.707
-0.703, 0.652, -1.000, -0.167

N = 5

V_1 = A, A, A, A, A, -0.618, 1.000, -0.090, -0.601, 0.999
-0.001, -0.600, 0.999, -0.000, -0.600, 0.999
0.000, -0.601, 0.999, 0.001, -0.618, 1.000
-0.090

Where A = -0.053, 0.463, -0.000, -0.192, 0.768, -0.000
-0.381, 0.930, -0.000, -0.593, 0.976, -0.000

V_2 = A, B, C, -B, -A, -0.640, -1.000, -0.139, -0.496, 0.800
-0.456, -0.189, 0.308, -0.734, 0.189, -0.309
-0.734, 0.496, -0.809, -0.456, -0.640, -1.000
-0.139

(b) First two eigen vectors calculated for coupled transverse torsional modes of banded blades N = 2 to 6 are given below for = 0.0385

N = 2

$V_1 \equiv$ A, A, -0.618, 1.00, -0.093, 0.601, 1.00, 0.00, -0.618
1.00, 0.09

Where A = 0.053, 0.046, -0.000, -0.192, 0.768, 0.000
-0.381, 0.930, -0.000, -0.598, 0.976, 0.000

$V_2 \equiv$ B, -B, 0.377, -0.427, 0.322, -0.000, 0.000, 1.000, -0.377
0.427, 0.322

Where B = 0.028, -0.239, 0.060, 0.096, -0.367, 0.111
0.183, -0.403, 0.145, 0.271, -0.395, 0.157

N = 3

V_1 = A, A, A, -0.618, 1.000, -0.09, -0.601, 1.000, -0.000
-0.601, 1.000, 0.000, -0.618, 1.000, -0.09

Where A = -0.053, 0.463, 0.000, -0.192, 0.768, 0.000
-0.381, 0.930, -0.000, -0.539, 0.967, - 0.000

V_2 = A, B, -A, 0.556, -0.819, 0.187, 0.260, -0.410, 1.000
-0.260, 0.410, 1.000, -0.556, 0.819, 0.187

Where A = 0.049, -0.422, -0.000, 0.173, -0.669, -0.000
0.333, -0.766, -0.001, 0.503, -0.774, -0.001

APPENDIX C

COMPUTER PROGRAM

This appendix gives the description of the computer programs used to calculate the natural frequencies of the banded group of blades. Two different programs, TANGEN and TRATOR are made for tangential and coupled Transverse - Torsional vibration modes. A listing of the programs and the subroutines used are added at the end.

C.1 Main Program TANGEN

This program calculates the natural frequencies of the tangential mode of vibration for the packetted steam turbine blades. The data required is to be defined in the program itself. Prints out the natural frequencies and the mode shapes. Only the natural frequencies are calculated if MODE = 0 is defined. Subroutines used are given below.

C.1.1 Name of Subroutine : STIFF

a) Argument list : A, D, M

A : Length of the element

D : Stiffness Matrix

M : Number of element

b) Common List : ZB1, E, R, W22

ZB1 : M.I. of the section about ZZ axis of the blade.

E : Youngs Modulus

R : Distance of the inner end A from the rotor axis.

W22 : This is square of the rotor speed.

c) Dimension Statement

D : (14, 14)

d) Purpose : To calculate the basic stiffness matrix of the matrix

C.1.2 Name of the Subroutine: MASS

a) Argument list : A, C, M

A, M : Defined as earlier

C : Mass Matrix of the element

b) Common Statement: RHO , H, AB

RHO : Mass per cubic inch = γ/g for blade

H : Elemental length

AB : Area of the blade cross section.

c) Dimension Statement

C : (14, 14)

d) Purpose : To establish the basic inertia matrix used in TANGEN

C.1.3 Name of the Subroutine: MATIN

a) Argument List : A, N

A : Matrix to be inverted, and the inverted matrix itself

N : Order of A

b) Common List		:	NIL
c) Dimension	A	:	(N, N)
d) Purpose		:	To invert the matrix.

C.1.4 Name of Subroutine : BAND

a) Argument list		:	E, RHO, I, J, K, L
	E, RHO	:	Youngs Module & specific mass density for the band.
	I, J, K, L	:	Co-ordinates of the band.
b) Common List		:	AM, AK
	AM	:	Mass Matrix of the whole system
	AK	:	Stiffness matrix of the whole system
c) Dimension	AM	:	The value given for MN, by equation 3.17
	AK	:	Same as AM
d) Purpose		:	To couple the blade matrices with those of the band

C.1.5 Name of the Subroutine : EIGMOD

a) Argument List		:	AK, AM, MN, MO, X, EIG
	AK, AM	:	Mentioned earlier
	MN	:	Order of matrix AK, AM
	MO	:	Number of lowest natural frequencies required.
	X	:	The matrix of mode shapes
	EIG	:	Natural frequencies

b) Common List : NIL

c) Dimension Statement:

AK, AM : (MN, MN)

X : (MN, MO)

EIG : (MO)

d) Purpose : To calculate the natural frequencies and the mode shapes.

C.1.6 Name of the Subroutine : SWEEP

a) Argument List : AK, AM, D, MN, III, X

AK, AM, MN : Mentioned earlier

D : Dynamical matrix

III : Number of lowest eigen value being calculated

b) Common List : NIL

c) Dimension Statement:

AK, AM : (MN, MN)

D : (MN, MN)

X : (MN, MO)

d) Purpose : To sweep the dynamical matrix by one dimension. This is called from EIGMOD.

C.2 Main Program : TRATOR

This program calculates the natural frequencies of the coupled Transverse - Torsional Modes of the banded turbine blades. The data of the blade specifications is to be defined in the programme itself.

C.2.1 Name of the Subroutine : STIFF

a) Argument List : A, D

A : Elemental length

D : Basic Stiffness Matrix

b) Common Statements : RHO, H, AB, XY; ZB2, E, R, W22, Y.

RHO, H, AB, E, R : Mentioned Earlier (for the blade element)

XY : Torsional inertia constant

Y : Torsional stiffness constant

ZB2 : M.I of blade section about the Y-Y axis.

c) Dimension Statement:

D : (6, 6)

d) Purpose : To calculate basic stiffness matrix

C.2.2 Name of the Subroutine : MASS

a) Argument list : A, C

A, C : Mentioned in C.1.2-a

b) Common list : RHO, H, AB, XY

RHO, H, AB, XY: Mentioned earlier

c) Dimension Statement:

C : (6.6)

C.2.3 Name of the Subroutine : BAND

a) Argument List : E, RHO, Y, XY, I1, I2, I3, J1, J2, J3

E, RHO, Y, XY : Mentioned Earlier

I1, I2, I3, J1, J2, J3 : Coordinate of the band element

b) Common Statement : AK, AM, S, ZP2, AP

AK, AM : Mentioned earlier

S : Band elemental length

ZP2 : M.I. of the band section about X-X axis.

AP : Cross-sectional area of the band.

c) Dimension Statement:

AK : (MN, MN)

AM : (MN, MN)

d) Purpose : To couple individual blade mass and stiffness matrices.

C.2.4 Name of the subroutine : MATIN

Described in C.1.4

C.2.5 Name of the subroutine : EIGEN

a) Argument list : AK, MN, MO, EIG

AK, MO, EIG : All mentioned earlier

MN : Order of matrix in the case of combined transverse-torsional mode as given in the text (Equation 3.18).

b) Common list : NIL

c) Dimension

AK : (MN, MN)

EIG : (MO)

d) Purpose : This calculates only the lower MO natural frequencies and no mode shapes.

```
$IBJOB
$IBFTC TRATOR
C
C*********************************************************************************
C          PROGRAM MADE BY, VASANT J. BHIDE ,  M.TECH.(MECH.)
C                                               I.I.T. KANPUR      7/8/1972
C*********************************************************************************
C
C
C     COUPLED TRANSVERSE AND TORSIONAL VIBRATIONS OF H. P. TURBINE BL
C
      DIMENSION AK(93,93),AM(93,93),AMB(18,18),AKB(18,18),STIF(6,6)
     *,AMASS(6,6),C(93),EIG(6),XXX(93,6)
      COMMON/AAA/RHO,H,AB,XY
      COMMON /BBB/ZB2,E,R,W22,Y
      COMMON /CCC/AK,AM,S,ZP1,AP
      CALL FLOV(32000)
      CALL FLUN(32000)
      MODE=0
C
C     PHYSICAL PROPRETIES OF THE  BLADE  AND THE BAND
C
      DATA E,ITMAX,G/30000000.,15,12000000./
      RHO=0.28/386.4
      A=0.88
      RO=17.5
      AB=212.857/654.2
      AP=2.667*26.162/654.2
      ZP1=26.162/25.4/12.*(2.667/25.4)**3
      ZP2=2.667/25.4/12.*(26.162/25.4)**3
      ZB2=5868.4/654.2/654.2
      S=19.301/25.4
      S=S/2.
      B=13.75/25.4
      D=17.35/25.4
      PRINT 10,A,B,D,AB,ZB2,AP,S,ZP1
      PRINT 11,E,G,RHO
CCC   B,D -EQUIVALENT BREADTH AND THICKNESS OF THE SECTION
      PI=3.141593
      W2=0.
      M=4
      AN=M
      H=A/AN
C
C     TORSIONAL CONSTANTS EVALUTED
      T=1.
      SUM=0.
  114 X=T*PI*D/B/2.
      MZ=0
  112 MZ=MZ+1
```

```
      X=X/2.
      IF(X.GT.4.) GO TO 112
      X=EXP(X)
      DO 113 I=1,MZ
113   X=X*X
      X=(X-1./X)/(X+1./X)
      X=X/T**5
      SUM=SUM+X
      T=T+2.
      IF(X.GT.0.0001) GO TO 114
      X=(1.-192./PI**5*B/D*SUM)/3.
      XJ=X*B**3*D
      Y=XJ*G/H
      XY=XJ*RHO*H/3.
      PRINT 12,Y,XY
CCC
C     BLADE STIFF. , MASS  MATRICES CALCULATED (--ST. 110 )
C     TO ACCOUNT EFFECT OF C. F. E,REMOVE CARD N/.... 104
CCC
      MM=3*M+3
      DO 100 I=1,MM
      DO 100 J=1,MM
      AKB(I,J)=0.
100   AMB(I,J)=0.
      W22=W2*W2
      DO 110 I=1,M
      AI=I-1
      I3=3*(I-1)
      R=RO+H*AI
104   IF(I.GT.1) GO TO 102
      CALL STIFF(H,STIF)
      CALL MASS(H,AMASS)
102   DO 110 J=1,6
      J3=I3+J
      DO 110 L=1,6
      L3=I3+L
      AKB(J3,L3)=AKB(J3,L3)+STIF(J,L)
110   AMB(J3,L3)=AMB(J3,L3)+AMASS(J,L)
CCC
C     BOUNDARY CONDITIONS APPLIED  (-- 120 )
      M3=MM-3
      DO 120 I=1,M3
      DO 120 J=1,M3
      AKB(I,J)=AKB(I+3,J+3)
120   AMB(I,J)=AMB(I+3,J+3)
C
      DO 2550 N=6,10
C
      RATIO=0.040
      IF(N.EQ.1) GO TO 111
```

```
280   ZP2=ZB2*RATIO
      D=SQRT(12.*ZP2/AP)
      B=AP/D
      ZP1=D*B**3/12.
      PRINT 11,B,D,AP,ZP1,ZP2
      T=1.
      SUM=0.
1114  X=T*PI*D/B/2.
      MZ=0
1112  MZ=MZ+1
      X=X/2.
      IF(X.GT.4.) GO TO 1112
      X=EXP(X)
      DO 1113 I=1,MZ
1113  X=X*X
      X=(X-1./X)/(X+1./X)
      X=X/T**5
      SUM=SUM+X
      T=T+2.
      IF(X.GT.0.0001) GO TO 1114
      X=(1.-192./PI**5*B/D*SUM)/3.
      XJ=X*B**3*D
      Y=XJ*G/H
      XY=XJ*RHO*H/3.
      PRINT 11,Y,XY
111   CONTINUE
CCC
      MN=3*M*N+3
      IF(N.NE.1) MN=MN+N+N+N
      DO 130 I=1,MN
      DO 130 J=1,MN
      AK(I,J)=0.
130   AM(I,J)=0.
CCC   ASSEMBLING OF MASS AND STIFNESS MATRUCES GOR THE SYSTEM (......1
      DO 140 I=1,N
      N3=3*M*(I-1)
      DO 140 J=1,M3
      NJ=J+N3
      DO 140 K=1,M3
      NK=K+N3
      AK(NJ,NK)=AKB(J,K)
140   AM(NJ,NK)=AMB(J,K)
CCC   COUPLING DUE TO BAND ( .............150 )
      IF(N.EQ.1) GO TO 160
      I1=3*M*N
      I2=I1-1
      I3=I2-1
      DO 150 II=1,N
      I1=I1+3
       I2=I2+3
```

```
      I3=I3+3
      J1=3*M*II
      J2=J1-1
      J3=J2-1
      CALL BAND (E,RHO,Y,XY,I1,I2,I3,J1,J2,J3)
      K1=J1
      K2=J2
      K3=J3
      L1=I1+3
      L2=I2+3
      L3=I3+3
      CALL BAND (E,RHO,Y,XY,K1,K2,K3,L1,L2,L3)
150   CONTINUE
160   CONTINUE
CCC   DYNAMICAL MATRIX FOR THE SYSTEM OBTAINED AT ST. N/.---170
      IF(N.EQ.1) MN=M3
      CALL MATIN (AK,MN)
      DO 170 I=1,MN
      DO 180 J=1,MN
      C(J)=0.
      DO 180 L=1,MN
180   C(J)=C(J)+AK(I,L)*AM(L,J)
      DO 170 L=1,MN
      C(L)=10000000000.*C(L)
170   AK(I,L)=C(L)
      PRINT 1,N,RATIO
      M2=6
C
      IF(MO) 84,84,85
85    CALL EIGMOD (AK,AM,MN,6,XXX,EIG)
      GO TO   86
84    CALL EIGEN (AK,MN,M2,EIG)
86    CONTINUE
C
      IF(N.EQ.1) GO TO 2550
      RATIO=RATIO+0.01
      IF(RATIO.LE.0.045) GO TO 280
255   CONTINUE
1     FORMAT (30X,*NO.OF.BLADES  = *I5/20X,*BLADE TO BAND STIFF. RATIO =
     1*,F10.5)
2     FORMAT (30X,*MODE SHAPE ---*/(1X,10E12.3))
3     FORMAT(10X,*NATURAL FREQUENCY --*,I5,*= *,E19.7)
4     FORMAT (1X,10E12.3)
10    FORMAT(/20X,*DATA---*/(1X,9E12.3))
11    FORMAT (1X,9E10.3)
12    FORMAT (/10X,*BLADE STIFFNESS AND INERTIA PROPERTIES*/15X,2E19.7)
13    FORMAT (/1X,*BLADE STIFFNESS MATRIX*/(1X,12E10.3))
14    FORMAT (/1X,*BLADE MASS       MATRIX*/(1X,12E10.3))
15    FORMAT (1X,*DYNAMICAL MATRIX*/(1X,12E10.3))
16    FORMAT (1X,12E10.3)
```

```
17      FORMAT (10X,* UNITY MATRUX *)
        STOP
        END
$IBFTC SUB1
        SUBROUTINE STIFF(A,D)
        COMMON/AAA/RHO,H,AB,XY
        COMMON /BBB/ZB2,E,R,W22,Y
        DIMENSION D(6,6)
        C=E
        B=ZB2
        M1=1
        M2=2
        M3=3
        M4=4
        M5=5
        M6=6
        DO 100 I=1,6
        DO 100 J=1,6
100     D(I,J)=0.
        D(M1,M1)=12.*E*B/A/A/A
        D(M2,M1)=-D(M1,M1)*A/2.
        D(M1,M2)=D(M2,M1)
        D(M4,M1)=-D(M1,M1)
        D(M1,M4)=D(M4,M1)
        D(M2,M2)=4.*E*B/A
        D(M4,M2)=-D(M2,M1)
        D(M2,M4)=D(M4,M2)
        D(M5,M2)=2.*E*B/A
        D(M2,M5)=D(M5,M2)
        D(M5,M1)=D(M2,41)
        D(M1,M5)=D(M5,M1)
        D(M4,M4)=D(M1,M1)
        D(M5,M4)=-D(M2,M1)
        D(M4,M5)=D(M5,M4)
        D(M5,M5)=D(M2,M2)
        D(M3,M3)=Y
        D(M3,M6)=-Y
        D(M6,M3)=-Y
        D(M6,M6)=Y
C
        PRINT 11,((D(J,K),K=1,6),J=1,6)
11      FORMAT (40X,*BASIC STIFFNESS MATRIX*/(10X,6E10.7))
111     RETURN
        END
$IBFTC SUB2
        SUBROUTINE MASS(A,C)
        COMMON/AAA/RHO,H,AB,XY
        DIMENSION C(6,6)
        B=AB
        M1=1
        M2=2
        M3=3
        M4=4
        M5=5
```

```
      M6=6
      DO 100 I=1,6
      DO 100 J=1,6
100   C(I,J)=0.
      R=RHO*AB*A
      C(M1,M1)=13.*R/35.
      C(M4,M4)=C(M1,M1)
      C(M4,M1)=9.*R/70.
      C(M1,M4)=C(M4,M1)
      C(M2,M1)=-11.*R*A/210.
      C(M1,M2)=C(M2,M1)
      C(M5,M4)=-C(M2,M1)
      C(M4,M5)=C(M5,M4)
      C(M2,M2)=R*A*A/105.
      C(M5,M5)=C(M2,M2)
      C(M5,M2)=-R*A*A/140.
      C(M2,M5)=C(M5,M2)
      C(M4,M2)=-C(M1,M5)
      C(M1,M5)=C(M5,M1)
      C(M5,M1)=13.*R*A/420.
      C(M2,M4)=C(M4,M2)
      C(M3,M3)=XY
      C(M6,M6)=XY
      C(M3,M6)=XY/2.
      C(M6,M3)=XY/2.
      PRINT 11,((C(I,J),J=1,6),I=1,6)
11    FORMAT (40X,*BASIC  INERTIA MATRIX*/(10X,6E19.7))
      RETURN
      END
$IBFTC SUB3
      SUBROUTINE MATIN (A,N)
      DIMENSION A(93,93)
      DO 640 I=1,N
      A(I,I)=1./A(I,I)
      DO 500 J=1,N
      IF(J-I) 600,500,600
600   A(I,J)=-A(I,J)*A(I,I)
500   CONTINUE
      DO 740 K=1,N
      IF(K-I) 800,740,800
800   DO 1020 J=1,N
      IF(J-I) 2000,1020,2000
2000   A(K,J)=A(K,J)+A(K,I)*A(I,J)
1020  CONTINUE
      A(K,I)=A(K,I)*A(I,I)
740   CONTINUE
640   CONTINUE
      RETURN
      END
$IBFTC SUB4
      SUBROUTINE BAND (E,RHO,Y,XY,I1,I2,I3,J1,J2,J3)
```

```
      DIMENSION AK(93,93),AM(93,93)
      COMMON /CCC/AK,AM,S,ZP2,AP
      X=E*ZP2/S
      AK(I1,I1)=AK(I1,I1)+4.*X
      AK(I3,I1)=AK(I3,I1)-6.*X/S
      AK(J1,I1)=AK(J1,I1)+2.*X
      AK(J3,I1)=AK(J3,I1)+6.*X/S
      AK(I2,I2)=AK(I2,I2)+Y
      AK(J2,I2)=AK(J2,I2)-Y
      AK(I1,I3)=AK(I1,I3)-6.*X/S
      AK(I3,I3)=AK(I3,I3)+12.*X/S/S
      AK(J1,I3)=AK(J1,I3)-6.*X/S
      AK(J3,I3)=AK(J3,I3)-12.*X/S/S
      AK(I1,J1)=AK(I1,J1)+2.*X
      AK(I3,J1)=AK(I3,J1)-6.*X/S
      AK(J1,J1)=AK(J1,J1)+4.*X
      AK(J3,J1)=AK(J3,J1)+6.*X/S
      AK(I2,J2)=AK(I2,J2)-Y
      AK(J2,J2)=AK(J2,J2)+Y
      AK(I1,J3)=AK(I1,J3)+6.*X/S
      AK(I3,J3)=AK(I3,J3)-12.*X/S/S
      AK(J1,J3)=AK(J1,J3)+6.*X/S
      AK(J3,J3)=AK(J3,J3)+12.*X/S/S
CCCCCC
      X=RHO*AP*S
      AM(I1,I1)=AM(I1,I1)+X*S*S/105.
      AM(I3,I1)=AM(I3,I1)-11.*X*S/210.
      AM(J1,I1)=AM(J1,I1)-X*S*S/140.
      AM(J3,I1)=AM(J3,I1)-13.*X*S/420.
      AM(J2,I2)=AM(J2,I2)+XY/2.
      AM(I2,I2)=AM(I2,I2)+XY
      AM(I1,I3)=AM(I1,I3)-11.*X*S/210.
      AM(I3,I3)=AM(I3,I3)+13.*X/35.
      AM(J1,I3)=AM(J1,I3)+13.*X*S/420.
      AM(J3,I3)=AM(J3,I3)+9.*X/70.
      AM(I1,J1)=AM(I1,J1)-X*S*S/140.
      AM(I3,J1)=AM(I3,J1)+13.*X*S/420.
      AM(J1,J1)=AM(J1,J1)+X*S*S/105.
      AM(J3,J1)=AM(J3,J1)+11.*X*S/210.
      AM(I2,J2)=AM(I2,J2)+XY/2.
      AM(J2,J2)=AM(J2,J2)+XY
      AM(I1,J3)=AM(I1,J3)-13.*X*S/420.
      AM(I3,J3)=AM(I3,J3)+9.*X/70.
      AM(J1,J3)=AM(J1,J3)+11.*X*S/210.
      AM(J3,J3)=AM(J3,J3)+13.*X/35.
      RETURN
      END
$IBFTC SUB6
      SUBROUTINE EIGEN (A,MN,MO,EIG)
      DIMENSION A(93,93),EIG(6),X(93),Y(93)
```

```
CCC   A-MATRIX,MN-ORDER,EIG-E.V.,MO-NO.OF VAL.
      PI=3.14159
      DO 1000 IV=1,MO
      ITMAX=30-2*IV
      N=MN-IV+1
      DO 100 I=1,N
100   Y(I)=1.
      ITERA=0
150   DO 110  J=1,N
      X(J)=0.
      DO 110 L=1,N
110   X(J)=X(J)+A(J,L)*Y(L)
      IF(ITERA.GT.10) GO TO 120
      M=1
      XMAX=ABS(X(1))
      DO 130 I=1,N
      IF(XMAX.GE.ABS(X(I))) GO TO 130
      XMAX=ABS(X(I))
      M=I
130   CONTINUE
120   CONTINUE
      R=X(M)
      DO 140 I=1,N
140   Y(I)=X(I)/R
      ITERA=ITERA+1
      IF(ITERA.LE.ITMAX) GO TO 150
      EIG(IV)=R
C
      SS=SQRT(1./R)/(2.*3.14159)
      SS=100000.*SS
      PRINT 17,SS
17 0  FORMAT (40X,E19.7)
C
      YMAX=Y(1)
      M=1
      DO 160 I=1,N
      IF(YMAX.GT.Y(I)) GO TO 160
      YMAX=Y(I)
      M=I
16 0  CONTINUE
      DO 170 I=1,N
170   X(I)=A(M,I)/(P*YMAX)
      DO 180 I=1,N
      DO 180 J=1,N
180   A(I,J)=A(I,J)-R*Y(I)*X(J)
      IF(M.EQ.N) GO TO 1000
      N1=N-1
      IF(N1.EQ.1) GO TO 220
      DO 200 I=1,N
      DO 200 J=M,N1
```

```
200     A(I,J)=A(I,J+1)
        DO 210 I=M,N1
        DO 210 J=1,N1
210     A(I,J)=A(I+1,J)
1       FORMAT (10X,E19.7)
100     CONTINUE
        GO TO 230
220     IF(M.EQ.1) EIG(MN)=A(2,2)
        IF(M.EQ.2) EIG(MN)=A(1,1)
230     RETURN
        END
        SUBROUTINE EIGMOD(AK,AM,MN,MO,X,EIG)
        DIMENSION AK(40,40),AM(40,40),X(40,6),D(40,40),Y(40),EIG(6)
        PI=3.1419
        E=0.0001
        DO 100 II=1,MO
        ITERA=0
        CALL SWEEP (AK,AM,D,MN,II,X)
        DO 110 I=1,MN
110     X(I,II)=1.
 99     KC=0
160     DO 120 I=1,MN
        Y(I)=0.
        DO 120 K=1,MN
120     Y(I)=Y(I)+D(I,K)*X(K,II)
        R=Y(1)
        DO 90 I=1,MN
        Y(I)=Y(I)/R
        IF(ABS(Y(I)/X(I,II)-1.0)-E) 90,90,102
102     KC=1
90      CONTINUE
        ITERA=ITERA+1
        IF(ITERA.GE.200) GO TO 1000
        IF(KC) 1000,1000,104
104     DO 115 I=1,MN
115     X(I,II)=Y(I)
        GO TO 99
1000     PRINT 1,R
1       FORMAT (10X,E19.7)
              EIG(II)=R
        R=SQRT(1./R)/(2.*PI)
        R=100000.*R
        PRINT 1,R
        PRINT 2,(X(J,II),J=1,MN)
2       FORMAT (1X,10E12.4)
100     CONTINUE
        RETURN
        END
$IBFTC SUB 11
        SUBROUTINE SWEEP(AK,AM,D,MN,III,X)
```

```
      DIMENSION AK(40,40),AM(40,40),D(40,40),X(40,6),ROR(6,40),SOR(6
      II=III-1
      IF(II) 111,111,112
112   DO 100 I=1,II
      DO 110 J=1,MN
      ROR(I,J)=0.
      DO 110 K=1,MN
110   ROR(I,J)=ROR(I,J)+AM(K,J)*X(K,I)
100   CONTINUE
      CALL MATIN(ROR,II)
      DO 130 J=1,II
      DO 130 I=III,MN
      SOR(J,I)=0.
      DO 130 K=1,II
130   SOR(J,I)=SOR(J,I)-ROR(J,K)*ROR(K,I)
      DO 140 I=1,MN
      DO 140 J=1,MN
      D(I,J)=0.
      IF(J-II) 140,140,141
141   DO 150 K=1,II
150    D(I,J)=D(I,J)+AK(I,K)*SOR(K,J)
      D(I,J)=D(I,J)+ AK(I,J)
140   CONTINUE
      GO TO 160
111   DO 170 I=1,MN
      DO 170 J=1,MN
170   D(I,J)=AK(I,J)
160   RETURN
      END
C
C
$IBJOB
$IBFTC TANGEN
C
C     MODE SHAPES FOR THE TANGENTIAL VIBRATIONS OF THE TURBINE BLADE
C
      DIMENSION AMASS(14,14),STIF(14,14),C(40),XXX(40,40)
      DIMENSION U(40),Y(40),XXX(40,6),EIG(6)
      CALL FLOV(32000)
      CALL FLUN(32000)
      COMMON /KANPUR /RHO,H,AB
      COMMON /POONA/ZB1,E,R,W22
      COMMON /DEHLI/S,AP,ZP1
      COMMON/B1/AM(40,40)
      COMMON/B2/AK(40,40)
      DATA E,ITMAX,RHO                              /29000000.,15,0.283
      MODE=1

      A=3.84
      R0=17.5
      AB=0.298
      AP=1.168*0.125
      ZB1=0.00771
```

```
      W2=0.
      RHO=RHO/386.4
      AN=M
      MM=2*M+2
      PI=3.1416
      DO 270 N=2,6
      X=0.02500000
      MN=2*M*N+2
      IF(N.EQ.1) GO TO 310
      MN=MN+N+N
300   CONTINUE
      ZP1=ZBI*X
      ZPI=1.168/12.*0.125**3
768   W22=W2*W2
C
310   DO 100 I=1,MN
      DO 100 J=1,MN
      AM(I,J)=0.
100   AK(I,J)=0.
      S=19.301/25.4
      S=0.790
      S=S/2.
      H=A/AN
C
      DO 110 I=1,M
      AI=I-1
      R=R0+H*AI
      II=2*I-1
      I3=II+3
      CALL STIFF (H,STIF,II)
      CALL MASS (H,AMASS,II)
      DO 110 J=II,I3
      DO 110 L=II,I3
      AK(J,L)=AK(J,L)+STIF(J,L)
11    AM(J,L)=AM(J,L)+AMASS(J,L)
C
      M2=MM-2
      DO 120 I=1,M2
      DO 120 J=1,M2
      AM(I,J)=AM(I+2,J+2)
120   AK(I,J)=AK(I+2,J+2)
      DO 130 I=1,MM
      DO 130 J=1,MM
      IF(I.LE.M2.AND.J.LE.M2) GO TO 130
140   AK(I,J)=0.
      AM(I,J)=0.
13 0  CONTINUE
C
C     MATRICES FOR THE REST OF THE BLADES ARE EQUATED -
C
      IF (N.EQ.1) GO TO 160
      N1=N-1
      DO 150 K=1,N1
      K2=2*M*K
      DO 150 I=1,M2
      DO 150 J=1,M2
```

```
      KI2=K2+I
      KJ2=K2+J
      AK(KI2,KJ2)=AK(I,J)
150   AM(KI2,KJ2)=AM(I,J)
C
C       ADDITION OF BAND STIFF. AND MASS  MATRICES DONE UPTO ST.NO. 17
C
      I=2*M*N
      J=I-1
      DO 170 II=1,N
      I=I+2
      J=J+2
      K=2*M*II
      L=K-1
      CALL BAND (E,RHO,I,J,K,L)
      I1=K
      J1=L
      K1=I+2
      L1=J+2
      CALL BAND(E,RHO,I1,J1,K1,L1)
170   CONTINUE
      IF(W2.EQ.0.) GO TO 160
C
C     STIFFNESS CORRECTION MADE TO ACCOUNT C. F. DUE TO  MASS OF THE
C
      DO 320 I=1,N
      J=2*M*I-1
320   AK(J,J)=AK(J,J)-2.*RHO*AP*S*W2*W2
160   CONTINUE
C
C     DYNAMICAL MATRIX OBTAINED AT ST.---770
C
      IF(N.EQ.1) MN=M2
      CALL MATIN (AK,MN)
      DO 770 I=1,MN
      DO 780 J=1,MN
      C(J)=0.
      DO 780 L=1,MN
780    C(J)=C(J)+AK(I,L)*AM(L,J)
      DO 770 L=1,MN
      C(L)=1000000000.*C(L)
770   AK(I,L)=C(L)
C
      PRINT 8,N,X
      IF(MODE) 84,84,85)
85    CALL EIGMOD (AK,AM,MN,6,XXX,EIG)
84    CALL EIGEN(AK,MN,6,EIGN)
86    CONTINUE
16    FORMAT (2X,6E19.7)
270   CONTINUE
1     FORMAT (8F10.5)
2     FORMAT(5I5)
3     FORMAT (/(5X,10F10.2))
4     FORMAT(/(1X,10E12.3))
5     FORMAT (10X,I7,5E19.7)
6     FORMAT (10X,10E12.3)
```

```
8     FORMAT (//10X,*NO. OF PACKETED BLADES = *,I10/10X,*BAND TO BLAD
     1STIFFNASS RATIO = *,E19.7)
11    FORMAT (30X,*REDUCED MATRICES *)
9     FORMAT (30X,*ASSEMBLED STIFF. AND MASS MATRICES* )
10    FORMAT (/(1X,12E10.3))
7     FORMAT(/10X,*FUNDAMENTAL NAT.FREQ. = *,I1,E19.7/(15X,*HIGHER NAT
     1REQ.*,I5,*  = *,E19.7))
14    FORMAT (20X,*ROTOR SPEED --* F10.5)
31    FORMAT(/(10X,8E13.5))
      STOP
      END
$IBFTC SUB1
      SUBROUTINE STIFF (A,D,M)
      COMMON /KANPUR /RHO,H,AB
      COMMON /POONA/ ZB1,E,R,W22
      DIMENSION D(14,14)
      C=E
      B=ZB1
C                      A  - LENGTH OF ELEMENTS
C                      B   - MOMENT OF INERTIA
C                      C  - YOUNG,S MODULUS
C                      M  - NUMBER OF ELEMENT
C                      D - STIFFNESS MATRIX
      D(M,M)=12.*C*B/(A*A*A)
      D(M+1,M)=D(M,M)*A/2.
      D(M,M+1)=D(M+1,M)
      D(M+2,M)=-D(M,M)
      D(M,M+2)=D(M+2,M)
      D(M+3,M)=D(M+1,M)
      D(M,M+3)=D(M+3,M)
      D(M+1,M+1)=4.*B*C/A
      D(M+3,M+3)=D(M+1,M+1)
      D(M+3,M+1)=D(M+1,M+1)/2.
      D(M+1,M+3)=D(M+3,M+1)
      D(M+3,M+2)=-D(M+3,M)
      D(M+2,M+3)=D(M+3,M+2)
      D(M+2,M+2)=D(M,M)
      D(M+1,M+2)=D(M+2,M+3)
      D(M+2,M+1)=D(M+1,M+2)
      IF(W22.EQ.0.) GO TO 100
      X=RHO*AB*AB*W22*A*A
      D(M,M)=D(M,M)-X*((R/2.+13./35.)/A+13./70.)
      D(M+1,M)=D(M+1,M)-X*A*((-R/10.+11./210.)/A+1./105.)
      D(M+2,M)=D(M+2,M)-X*((-R/2.+9./70.)/A-13./70.)
      D(M+3,M)=D(M+3,M)-X*A*((R/10.-13./420.)/A+11./420.)
      D(M,M+1)=D(M,M+1)-X*A*((R/10.+11./210.)/A+3./70.)
      D(M+1,M+1)=D(M+1,M+1)-X*A*A*( 1./210.+1./(105.*A))
      D(M+2,M+1)=D(M+2,M+1)-X*A*((-R/10.+13./420.)/A-3./70.)
      D(M+3,M+1)=D(M+3,M+1)-X*A*A*((R/60.-1./170.)/A+1./210.)
      D(M,M+2)=D(M,M+2)-X*((R/2.+9./70.)/A+11./35.)
```

```
      D(M+1,M+2)=D(M+1,M+2)-X*A*((R/10.+13./420.)/A+31./420.)
      D(M+2,M+2)=D(M+2,M+2)-X*((-R/2.+13./35.)/A-11./35.)
      D(M+3,M+2)=D(M+3,M+2)-X*A*((R/10.-11./210.)/A-23./210.)
      D(M,M+3)=D(M,M+3)-X*A*((-R/10.+13./420.)/A-2./35.)
      D(M+1,M+3)=D(M+1,M+3)-X*A*A*((-R/60.-1./140.)/A-1./84.)
      D(M+2,M+3)=D(M+2,M+3)-X*A*((R/10.-11./210.)/A+2./35.)
      D(M+3,M+3)=D(M+3,M+3)-X*A*A*(1./210.+1./(105.*A))
100   RETURN
      END
$IBFTC SUB4
      SUBROUTINE MASS (A,C,M)
      COMMON / KANPUR / RHO,H,AB
      DIMENSION C(14,14)
C        R - SPECIFIC DENSITY
C        A - LENGTH OF ELEMENT
C        B - AREA
C        C - MASS MATRIX
      B=AB
      R=RHO
      C(M,M)=156.*R*A*B/420.
      C(M+2,M+2)=C(M,M)
      C(M+2,M)=54.*R*B*A/420.
      C(M,M+2)=C(M+2,M)
      C(M+1,M)=22.*R*A*A*B/420.
      C(M,M+1)=C(M+1,M)
      C(M+3,M+2)=-C(M+1,M)
      C(M+2,M+3)=C(M+3,M+2)
      C(M+1,M+1)=R*B*A**3/105.
      C(M+3,M+3)=C(M+1,M+1)
      C(M+3,M+1)=-R*B*A**3/140.
      C(M+1,M+3)=C(M+3,M+1)
      C(M+3,M)=-13.*R*B*A*A/420.
      C(M+2,M+1)=-C(M+3,M)
      C(M+1,M+2)=-C(M+3,M)
      C(M ,M+3 )=C(M+3,M)
      RETURN
      END
$IBFTC SUB5
      SUBROUTINE BAND (E,RHO,I,J,K,L)
      COMMON/B1/AM(40,40)
      COMMON/B2/AK(40,40)
      COMMON /DEHLI/S,AP,ZP1
      AK(K,K)=AK(K,K)+4.*E*ZP1/S
      AK(I,I)=AK(I,I)+4.*E*ZP1/S
      AK(K,I)=AK(K,I)+2.*E*ZP1/S
      AK(I,K)=AK(I,K)+2.*E*ZP1/S
      AK(L,L)=AK(L,L)+AP*E/S
      AK(J,J)=AK(J,J)+AP*E/S
      AK(J,L)=AK(J,L)-AP*E/S
      AK(L,J)=AK(L,J)-AP*E/S
      XJ=RHO*AP*S
      AM(K,K)=AM(K,K)+XJ*S*S/105.
```

```
AM(I,I)=AM(I,I)+XJ*S*S/105.
AM(I,K)=AM(I,K)-XJ*S*S/140.
AM(K,I)=AM(K,I)-XJ*S*S/140.
AM(L,L)=AM(L,L)+XJ/3.
AM(J,J)=AM(J,J)+XJ/3.
AM(J,L)=AM(J,L)+XJ/6.
AM(L,J)=AM(L,J)+XJ/6.
RETURN
END
```

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